Interactive Fuzzy Programming based on Simple Recourse for Two-Level Integer Programming Problems with Random Variables

Takeshi Matsui[1, a, *], Antonio Oliveira Nzinga Rene[2,b] and Koji Okuhara[2,c]

[1] Faculty of Informatics, Gunma University, 4-2 Aramaki-machi, Maebashi, Gunma, 371-8510, Japan

[2] Faculty of Engineering, Toyama Prefectural University, 5180 Kurokawa, Imizu, Toyama 939-0398, Japan

* Corresponding author

[a]<tak-matsui@gunma-u.ac.jp>, [b]<rene@pu-toyama.ac.jp >, [c]<okuhara@pu-toyama.ac.jp>

Keywords: two-level integer programming, random variables, simple recourse model, interactive fuzzy programming

Abstract. This paper considers the optimization of hierarchical systems where two decision makers with different priority exist in stochastic environments. To be more specific, formulating them as two-level integer programming problems where right-hand constants in constraints are random variables, we reformulate them as two-level simple recourse problem. For these reformulated problems, we attempt to apply interactive fuzzy programming in order to derive satisfactory solutions for the decision maker at the upper level in consideration of the balance between the satisfactory level to the decision maker at the upper level and that to the decision maker at the lower level.

1. Introduction

In the real world, we often encounter situations where there are two decision makers (DMs) in an organization with a hierarchical structure, and they make decisions in turn or at the same time so as to optimize their objective functions. Such decision making situations can be formulated as a two-level programming problem [1]; one of the DMs first makes a decision, and then the other who knows the decision of the opponent makes a decision. Meanwhile, in actual decision making situations, some stochastic events may influence elements characterizing decision making problems such as demands of products, the amount of available resources and so forth. For such decision making problems involving uncertainty, there exist two typical approaches: probability theoretic approach [2, 3] and fuzzy-theoretic one [3-5]. Stochastic programming, as an optimization method based on the probability theory, have been developing in various ways [2, 3], including two stage problems considered by Dantzig [6] and chance constrained programming proposed by Charnes et al. [7]. Fuzzy mathematical programming representing the vagueness in decision making situations by fuzzy concepts have been studied by many researchers [8, 9]. Fuzzy two-level linear programming have been also developed by numerous researchers, and an increasing number of successful applications has been appearing [3, 8, 9]. When such a decision making problem under uncertainty is formulated as a linear programming problem, it may be difficult that the constraints of the problem always hold completely. Then, a shortage or an excess comes from the violation of the constraints, and the corresponding penalties are imposed as the occasion demands. From this point of view, a simple recourse model [6] had been investigated. In particular, after reformulating stochastic two-level linear programming problems, Kato et al. [10] presented an interactive fuzzy programming method to derive a satisfactory solution for the DM as a generalization of their previous results. Furthermore, it is often found that in real-world decision making situations, decision variables in a stochastic programming problem are not continuous but rather discrete. From this observation, we discuss interactive fuzzy stochastic two-level integer programming which is a natural extension of stochastic two-level linear programming with continuous variables discussed in [10].

Under these circumstances, in this paper, assuming cooperative behavior of the DMs, we consider solution methods based on a simple recourse model for stochastic two-level integer programming problems. Interactive fuzzy programming to obtain a satisfactory solution for the DM at the upper level in consideration of the cooperative relation between the DMs is presented.

2. Stochastic Two-Level Integer Programming

We deal with two-level integer programming problems involving random variable coefficients in the right-hand side of constraints formulated as

$$\begin{array}{ll} \underset{\text{for DM1}}{\text{minimize}} & z_1(\boldsymbol{x}_1, \boldsymbol{x}_2) = \boldsymbol{c}_{11}\boldsymbol{x}_1 + \boldsymbol{c}_{12}\boldsymbol{x}_2 \\ \underset{\text{for DM2}}{\text{minimize}} & z_2(\boldsymbol{x}_1, \boldsymbol{x}_2) = \boldsymbol{c}_{21}\boldsymbol{x}_1 + \boldsymbol{c}_{22}\boldsymbol{x}_2 \\ \text{subject to} & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 = \boldsymbol{b}(\omega) \\ & x_{1j_1} \in \{0, 1, \dots, v_{1j_1}\}, j_1 = 1, \dots, n_1 \\ & x_{2j_2} \in \{0, 1, \dots, v_{2j_2}\}, j_2 = 1, \dots, n_2 \end{array} \tag{1}$$

where $\boldsymbol{x}_1$ is an n_1 dimensional decision variable column vector for the DM at the upper level, $\boldsymbol{x}_2$ is an n_2 dimensional decision variable column vector for the DM at the lower level, $A_j, j = 1, 2$ are $m \times n_j$ coefficient matrices, $\boldsymbol{c}_{lj}, l = 1, 2$ are n_j dimensional coefficient row vectors, and $\boldsymbol{b}(\omega)$ is an m dimensional column vector whose elements are independent random variables with continuous and nondecreasing probability distribution. For notational convenience, let DM1 and DM2 denote the DMs at the upper and the lower levels, respectively, and "$\underset{\text{for DM1}}{\text{minimize}}$" and "$\underset{\text{for DM2}}{\text{minimize}}$" mean that DM1 and DM2 are minimizers for their objective functions.

3. Two-Level Simple Recourse Programming

In the chance constrained problems, for random data variations, a mathematical model is formulated such that the violation of the constraints is permitted up to specified probability levels. On the other hand, in a two-stage model including a simple recourse model as a special case, a shortage or an excess arising from the violation of the constraints is penalized, and then the expectation of the amount of the penalties for the constraint violation is minimized.

It is assumed that a general case of the recourse model, the DM must make a decision before the realized values of the random variables involved in (1) are observed, and the penalty of the violation of the constraints is incorporated into the objective function in order to consider the loss caused by random data variations.

To be more specific, by expressing the difference between $A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2$ and $\boldsymbol{b}(\omega)$ in (1) as two vectors $\boldsymbol{y}^+ = (y_1^+, \dots, y_m^+)^T$ and $\boldsymbol{y}^- = (y_1^-, \dots, y_m^-)^T$, the expectation of a recourse for the lth objective function is represented by

$$R_l(\boldsymbol{x}) = E\left[\min_{\boldsymbol{y}^+, \boldsymbol{y}^-} (\boldsymbol{p}_l\boldsymbol{y}^+ + \boldsymbol{q}_l\boldsymbol{y}^-)\right]$$

where $\boldsymbol{p}_l$ and $\boldsymbol{q}_l$ are m dimensional constant row vectors, and $\boldsymbol{b}(\omega)$ is an m dimensional realization vector of $\boldsymbol{b}$ for an elementary event ω. Thinking of each element of $\boldsymbol{y}^+ = (y_1^+, \dots, y_m^+)^T$ and $\boldsymbol{y}^- = (y_1^-, \dots, y_m^-)^T$ as a shortage and an excess of the left-hand side, respectively, we can regard each element of $\boldsymbol{p}_l$ and $\boldsymbol{q}_l$ as the cost to compensate the shortage and the cost to dispose the excess, respectively.

Then, for the stochastic two-level integer programming problem, the simple recourse problem is formulated as

$$\begin{array}{ll} \underset{\text{for DM1}}{\text{minimize}} & z_1'(\boldsymbol{x}_1, \boldsymbol{x}_2) = \boldsymbol{c}_{11}\boldsymbol{x}_1 + \boldsymbol{c}_{12}\boldsymbol{x}_2 + R_1(\boldsymbol{x}) \\ \underset{\text{for DM2}}{\text{minimize}} & z_2'(\boldsymbol{x}_1, \boldsymbol{x}_2) = \boldsymbol{c}_{21}\boldsymbol{x}_1 + \boldsymbol{c}_{22}\boldsymbol{x}_2 + R_2(\boldsymbol{x}) \\ \text{subject to} & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 + \boldsymbol{y}^+ - \boldsymbol{y}^- = \boldsymbol{b}(\omega) \\ & x_{1j_1} \in \{0, 1, \dots, v_{1j_1}\}, j_1 = 1, \dots, n_1 \\ & x_{2j_2} \in \{0, 1, \dots, v_{2j_2}\}, j_2 = 1, \dots, n_2 \\ & \boldsymbol{y}^+ \geq \boldsymbol{0}, \boldsymbol{y}^- \geq \boldsymbol{0}. \end{array} \tag{2}$$

Because $\boldsymbol{p}_l$ and $\boldsymbol{q}_l$ are interpreted as penalty coefficients for shortages and excesses, it is quite natural to assume that $\boldsymbol{p}_l \geq \boldsymbol{0}$ and $\boldsymbol{q}_l \geq \boldsymbol{0}$, and then, it is evident that, for all $i = 1, \dots, m$, the complementary relations

$$y_i^+ > 0 \rightarrow y_i^- = 0, y_i^- > 0 \rightarrow y_i^+ = 0$$

should be satisfied for an optimal solution. With this observation in mind, we have

$$y_i^+ = b_i - \boldsymbol{a}_{i1}\boldsymbol{x}_1 - \boldsymbol{a}_{i2}\boldsymbol{x}_2, y_i^- = 0 \text{ if } b_i > \boldsymbol{a}_{i1}\boldsymbol{x}_1 + \boldsymbol{a}_{i2}\boldsymbol{x}_2$$
$$y_i^+ = 0, y_i^- = 0 \text{ if } b_i = \boldsymbol{a}_{i1}\boldsymbol{x}_1 + \boldsymbol{a}_{i2}\boldsymbol{x}_2$$
$$y_i^+ = 0, y_i^- = \boldsymbol{a}_{i1}\boldsymbol{x}_1 + \boldsymbol{a}_{i2}\boldsymbol{x}_2 - b_i \text{ if } b_i < \boldsymbol{a}_{i1}\boldsymbol{x}_1 + \boldsymbol{a}_{i2}\boldsymbol{x}_2.$$

Recalling that b_i are mutually independent. (2) can be explicitly calculated as

$$\begin{aligned} R_l(\boldsymbol{x}) &= E\left[\min_{\boldsymbol{y}^+, \boldsymbol{y}^-}(\boldsymbol{p}_l\boldsymbol{y}^+ + \boldsymbol{q}_l\boldsymbol{y}^-)\right] = \sum_{i=1}^m p_{li}\int_{\boldsymbol{a}_i\boldsymbol{x}}^{+\infty}(b_i - \boldsymbol{a}_i\boldsymbol{x})dF_i(b_i) + \sum_{i=1}^m q_{li}\int_{-\infty}^{\boldsymbol{a}_i\boldsymbol{x}}(\boldsymbol{a}_i\boldsymbol{x} - b_i)dF_i(b_i) \\ &= \sum_{i=1}^m p_{li}E[b_i] - \sum_{i=1}^m (p_{li} + q_{li})\int_{-\infty}^{\boldsymbol{a}_i\boldsymbol{x}} b_i dF_i(b_i) - \sum_{i=1}^m p_{li}\boldsymbol{a}_i\boldsymbol{x} + \sum_{i=1}^m (p_{li} + q_{li})(\boldsymbol{a}_i\boldsymbol{x})F_i(\boldsymbol{a}_i\boldsymbol{x}) \end{aligned}$$

where $\boldsymbol{x}$ denotes the concatenation of $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ and that $\boldsymbol{a}_i$ denotes the ith row of the matrix obtained from A_1 and A_2 by concatenating them in the horizontal direction, and F_i is the probability distribution function of b_i.

Then, (2) can be rewritten as

$$\begin{array}{ll} \underset{\text{for DM1}}{\text{minimize}} & Z_1(\boldsymbol{x}_1, \boldsymbol{x}_2) \\ \underset{\text{for DM2}}{\text{minimize}} & Z_2(\boldsymbol{x}_1, \boldsymbol{x}_2) \\ \text{subject to} & x_{1j_1} \in \{0, 1, \dots, v_{1j_1}\}, j_1 = 1, \dots, n_1 \\ & x_{2j_2} \in \{0, 1, \dots, v_{2j_2}\}, j_2 = 1, \dots, n_2 \\ & \boldsymbol{y}^+ \geq \boldsymbol{0}, \boldsymbol{y}^- \geq \boldsymbol{0}. \end{array} \tag{3}$$

where

$$\begin{aligned} Z_l(\boldsymbol{x}_1, \boldsymbol{x}_2) = \sum_{i=1}^m p_{li}E[b_i] &+ \sum_{j=1}^{n_1}\left(c_{l1j} - \sum_{i=1}^m a_{i1j}p_{li}\right)x_{1j} + \sum_{j=1}^{n_2}\left(c_{l2j} - \sum_{i=1}^m a_{i2j}p_{li}\right)x_{2j} \\ &+ \sum_{i=1}^m (p_{li} + q_{li}) \times \left\{(\boldsymbol{a}_{i1}\boldsymbol{x}_1 + \boldsymbol{a}_{i2}\boldsymbol{x}_2)F_i(\boldsymbol{a}_{i1}\boldsymbol{x}_1 + \boldsymbol{a}_{i2}\boldsymbol{x}_2) - \int_{-\infty}^{(\boldsymbol{a}_{i1}\boldsymbol{x}_1 + \boldsymbol{a}_{i2}\boldsymbol{x}_2)} b_i dF_i(b_i)\right\}. \end{aligned}$$

It should be noted here that (3) is a convex programming problem due to the convexity of $Z_l(\boldsymbol{x}_1, \boldsymbol{x}_2)$, and this means that (3) can be solved by using a conventional convex programming technique such as the sequential quadratic programming method [11].

4. Interactive Fuzzy Programming

In order to consider the imprecise nature of the DMs' judgments for each objective functions $Z_l(\boldsymbol{x}_1, \boldsymbol{x}_2)$ in (3), we introduce a fuzzy goal such as "$Z_l(\boldsymbol{x}_1, \boldsymbol{x}_2)$ should be substantially less than or equal to a certain value," (3) can be interpreted as

$$\begin{array}{ll}\underset{\text{for DM1}}{\text{maximize}} & \mu_1\big(Z_1(\boldsymbol{x}_1, \boldsymbol{x}_2)\big) \\ \underset{\text{for DM2}}{\text{maximize}} & \mu_2\big(Z_2(\boldsymbol{x}_1, \boldsymbol{x}_2)\big) \\ \text{subject to} & x_{1j_1} \in \{0, 1, \dots, v_{1j_1}\}, j_1 = 1, \dots, n_1 \\ & x_{2j_2} \in \{0, 1, \dots, v_{2j_2}\}, j_2 = 1, \dots, n_2 \\ & \boldsymbol{y}^+ \geq \boldsymbol{0}, \boldsymbol{y}^- \geq \boldsymbol{0},\end{array} \tag{4}$$

where μ_l is a membership function to quantify a fuzzy goal for the DMl in (4) as shown in Fig.1.

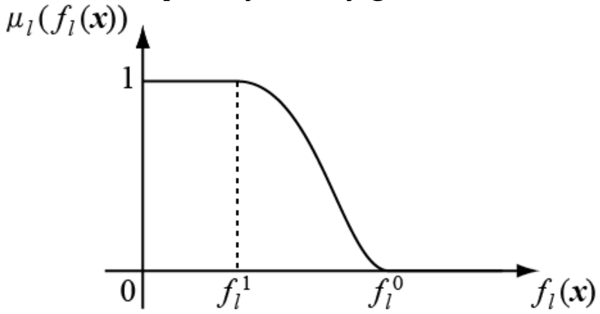

Fig.1 Example of a membership function $\mu_l(f_l(\boldsymbol{x}))$

As an initial candidate for an overall satisfactory solution to the DMs, it would be useful for DM1 to obtain a solution which maximizes the smaller degree of satisfaction between the two DMs by solving the maximin problem

$$\begin{array}{ll}\text{maximize} & \min\{\mu_1\big(Z_1(\boldsymbol{x}_1, \boldsymbol{x}_2)\big), \mu_2\big(Z_2(\boldsymbol{x}_1, \boldsymbol{x}_2)\big)\} \\ \text{subject to} & x_{1j_1} \in \{0, 1, \dots, v_{1j_1}\}, j_1 = 1, \dots, n_1 \\ & x_{2j_2} \in \{0, 1, \dots, v_{2j_2}\}, j_2 = 1, \dots, n_2 \\ & \boldsymbol{y}^+ \geq \boldsymbol{0}, \boldsymbol{y}^- \geq \boldsymbol{0}\end{array} \tag{5}$$

or equivalently

$$\begin{array}{ll}\text{maximize} & v \\ \text{subject to} & \mu_1\big(Z_1(\boldsymbol{x}_1, \boldsymbol{x}_2)\big) \geq v \\ & \mu_2\big(Z_2(\boldsymbol{x}_1, \boldsymbol{x}_2)\big) \geq v \\ & x_{1j_1} \in \{0, 1, \dots, v_{1j_1}\}, j_1 = 1, \dots, n_1 \\ & x_{2j_2} \in \{0, 1, \dots, v_{2j_2}\}, j_2 = 1, \dots, n_2 \\ & \boldsymbol{y}^+ \geq \boldsymbol{0}, \boldsymbol{y}^- \geq \boldsymbol{0}.\end{array} \tag{6}$$

If DM1 is satisfied with the membership function values $\mu_l\big(Z_l(\boldsymbol{x}_1, \boldsymbol{x}_2)\big)$ of (6), it follows that the corresponding optimal solution $(\boldsymbol{x}_1^*, \boldsymbol{x}_2^*)$ becomes a satisfactory solution; however, DM1 is not always satisfied with the membership function values. It is quite natural to assume that DM1 specifies the minimal satisfactory level $\hat{\delta} \in (0, 1]$ for the membership function $\mu_1\big(Z_1(\boldsymbol{x}_1, \boldsymbol{x}_2)\big)$ subjectively. Consequently, if DM1 is not satisfied with $\mu_l\big(Z_l(\boldsymbol{x}_1, \boldsymbol{x}_2)\big)$ of (6), the following problem is formulated:

$$\begin{array}{ll} \text{maximize} & \mu_2\big(Z_2(\boldsymbol{x}_1, \boldsymbol{x}_2)\big) \\ \text{subject to} & \mu_1\big(Z_1(\boldsymbol{x}_1, \boldsymbol{x}_2)\big) \geq \hat{\delta} \\ & x_{1j_1} \in \{0, 1, \ldots, v_{1j_1}\}, j_1 = 1, \ldots, n_1 \\ & x_{2j_2} \in \{0, 1, \ldots, v_{2j_2}\}, j_2 = 1, \ldots, n_2 \\ & \boldsymbol{y}^+ \geq \boldsymbol{0}, \boldsymbol{y}^- \geq \boldsymbol{0}, \end{array} \tag{7}$$

where DM2's membership function is maximized under the condition that DM1's membership function $\mu_1\big(Z_1(\boldsymbol{x}_1, \boldsymbol{x}_2)\big)$ is larger than or equal to the minimal satisfactory level $\hat{\delta}$ specified by DM1.

If there exists an optimal solution to (7), it follows that DM1 obtains a satisfactory solution having a satisfactory degree larger than or equal to the minimal satisfactory level specified by DM1. However, it is significant to realize that the larger the minimal satisfactory level $\hat{\delta}$ for μ_1 is accessed, the smaller the DM's satisfactory degree μ_2 becomes when the objective functions of DM1 and DM2 conflict with each other. Consequently, a relative difference between the satisfactory degrees of DM1 and DM2 becomes larger, and it follows that the overall satisfactory balance between both DMs is not appropriate.

In order to take account of the overall satisfactory balance between both DMs, realizing that DM1 needs to compromise with DM2 on DM1's own minimal satisfactory level, we introduce the ratio Δ of the satisfactory degree of DM2 to that of DM1 defined as

$$\Delta = \frac{\mu_2\big(Z_2(\boldsymbol{x}_1, \boldsymbol{x}_2)\big)}{\mu_1\big(Z_1(\boldsymbol{x}_1, \boldsymbol{x}_2)\big)}. \tag{8}$$

DM1 is guaranteed to have a satisfactory degree larger than or equal to the minimal satisfactory level for the fuzzy goal. To take into account the overall satisfactory balance between both DMs. DM1 specifies the lower bound $\Delta_{\min}$ and the upper bound $\Delta_{\max}$ for the ratio, and the ratio Δ is evaluated by verifying whether or not it is in the interval $[\Delta_{\min}, \Delta_{\max}]$. This condition is represented by $\Delta \in [\Delta_{\min}, \Delta_{\max}]$.

Now we are ready to present a procedure of interactive fuzzy programming for the simple recourse model for deriving an overall satisfactory solution.

[Interactive fuzzy programming in the simple recourse model]

Step 1: Calculate the individual minima $Z_{l,\min}$ and maxima $Z_{l,\max}$ of $Z_l(\boldsymbol{x}_1, \boldsymbol{x}_2), l = 1, 2$ by solving the integer programming problems.

Step 2: Ask each DM to specify the membership function μ_l by considering the individual minima and maxima obtained in step 1.

Step 3: Ask DM1 to specify the lower bound $\Delta_{\min}$ and the upper bound $\Delta_{\max}$ for the ratio Δ defined by (8).

Step 4: Solve the maximin problem (5), and calculate the membership function values $\mu_l\big(Z_l(\boldsymbol{x}_1, \boldsymbol{x}_2)\big), l = 1, 2$ and the ratio Δ corresponding to the optimal solution $(\boldsymbol{x}_1^*, \boldsymbol{x}_2^*)$ to (5). If DM1 is satisfied with the current membership function values, then stop. Otherwise, ask DM1 to specify the minimal satisfactory level $\hat{\delta} \in (0, 1]$ for the membership function $\mu_1\big(Z_1(\boldsymbol{x}_1, \boldsymbol{x}_2)\big)$.

Step 5: For the current minimal satisfactory level $\hat{\delta}$, solve the problem (7), and calculate the corresponding membership function values $\mu_l\big(Z_l(\boldsymbol{x}_1, \boldsymbol{x}_2)\big), l = 1, 2$ and the ratio Δ.

Step 6: If DM1 is satisfied with the membership function values $\mu_l\big(Z_l(\boldsymbol{x}_1, \boldsymbol{x}_2)\big), l = 1, 2$ and $\Delta \in [\Delta_{\min}, \Delta_{\max}]$ holds, then stop. Otherwise, ask DM1 to update the minimal satisfactory level $\hat{\delta}$, and return to step 5.

5. Numerical example

To demonstrate the feasibility and efficiency of the presented interactive fuzzy programming for the simple recourse model, consider the following stochastic two-level integer programming problem:

$$\begin{array}{ll} \underset{\text{for DM1}}{\text{minimize}} & z_1(\boldsymbol{x}_1, \boldsymbol{x}_2) = \boldsymbol{c}_{11}\boldsymbol{x}_1 + \boldsymbol{c}_{12}\boldsymbol{x}_2 \\ \underset{\text{for DM2}}{\text{minimize}} & z_2(\boldsymbol{x}_1, \boldsymbol{x}_2) = \boldsymbol{c}_{21}\boldsymbol{x}_1 + \boldsymbol{c}_{22}\boldsymbol{x}_2 \\ \text{subject to} & \boldsymbol{a}_{i1}\boldsymbol{x}_1 + \boldsymbol{a}_{i2}\boldsymbol{x}_2 = b_i(\omega), i = 1, 2, 3 \\ & x_{1j_1} \in \{0, 1, \dots, v_{1j_1}\}, j_1 = 1, \dots, 5 \\ & x_{2j_2} \in \{0, 1, \dots, v_{2j_2}\}, j_2 = 1, \dots, 5 \end{array} \tag{9}$$

where $b_1(\omega), b_2(\omega)$ and $b_3(\omega)$ are Gaussian random variables $N(230, 3^2), N(345, 4^2)$ and $N(437, 5^2)$, respectively; the coefficient vectors $\boldsymbol{c}_l, l = 1, 2,$ and $\boldsymbol{a}_i, i = 1, 2, 3$ are shown as in Tables 1. The constant row vectors $\boldsymbol{p}_l$ and $\boldsymbol{q}_l, l = 1, 2$ for the recourse variable vectors $\boldsymbol{y}^+$ and $\boldsymbol{y}^-$ are given in Table 2. The individual minima $Z_{l,\min}$ and maxima $Z_{l,\max}$ of $Z_l(\boldsymbol{x}_1, \boldsymbol{x}_2), l = 1, 2$ are calculated by solving the problems. Taking account of these values, assume that each DM determines the linear membership function by using Zimmermann's method [4].

Table 1. Each element of $\boldsymbol{c}_l, l = 1, 2,$ and $\boldsymbol{a}_i, i = 1, 2, 3$

$\boldsymbol{c}_{1j}$	3	5	2	6	1	1	4	7	2	9
$\boldsymbol{c}_{2j}$	−8	−1	−2	−7	−3	−5	−1	−4	−10	−5
$\boldsymbol{a}_{1j}$	4	4	1	2	6	1	1	7	5	3
$\boldsymbol{a}_{2j}$	10	2	6	1	2	2	8	5	2	8
$\boldsymbol{a}_{3j}$	3	8	8	5	1	9	7	7	3	2

Table 2. Each element of $\boldsymbol{p}_l$ and $\boldsymbol{q}_l, l = 1, 2$

$\boldsymbol{p}_1$	2.0	0.4	0.4	$\boldsymbol{q}_1$	0.2	0.6	0.3
$\boldsymbol{p}_2$	1.2	1.0	0.6	$\boldsymbol{q}_2$	1.4	0.9	1.1

For the upper bound $\Delta_{\max} = 0.900$ and the lower bound $\Delta_{\min} = 0.700$ specified by DM1, the maximin problem (5) is solved, and DM1 is supplied with the corresponding membership function values $\mu_l(Z_l(\boldsymbol{x}_1, \boldsymbol{x}_2))$ and the ratio Δ of the first iteration as shown in Table 3.

Table 3. Process of interaction

Iteration	1st	2nd
$\hat{\delta}$	−	0.55
$Z_2(\boldsymbol{x}_1, \boldsymbol{x}_2)$	306.716	292.734
$Z_2(\boldsymbol{x}_1, \boldsymbol{x}_2)$	−557.129	−530.395
$\mu_1(Z_1(\boldsymbol{x}_1, \boldsymbol{x}_2))$	0.543	0.583
$\mu_2(Z_2(\boldsymbol{x}_1, \boldsymbol{x}_2))$	0.543	0.474
Δ	1.000	0.813

Assume that DM1 is not satisfied with the membership function values, and DM1 specifies the minimal satisfactory level $\hat{\delta}$ for $\mu_1(Z_1(\boldsymbol{x}_1, \boldsymbol{x}_2))$. For the specified $\hat{\delta}$, the corresponding membership function values and the ratio of the second iteration are calculated.

Then, if DM1 is satisfied with the membership function values of the second iteration, it follows that a satisfactory solution is obtained.

6. Conclusion

In this paper, we focused on two-level integer programming problems with random variables in in the right-hand side of the constraints. Through the use of the simple recourse model, the formulated stochastic two-level integer programming problems with simple recourse are transformed into deterministic ones. An illustrative numerical example was provided to demonstrate the feasibility and efficiency of the proposed method. Extensions to other stochastic programming models will be considered elsewhere.

Acknowledgements

This work was supported by JSPS KAKENHI Grant Number JP 20K04981.

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