

# FE model for loosely-supported setup of impedance tube measurement

Yoshio Kurosawa<sup>1,a</sup>

<sup>1</sup>Faculty of Science and Technology, Teikyo University, 1-1 Toyosatodai, Utsunomiya City 320-8551, Japan

<sup>a</sup><ykurosawa@mps.teikyo-u.ac.jp>

**Keywords:** sound absorption, FEM, impedance tube, soundproof material

**Abstract.** In impedance tube measurement, the sound absorption coefficient measurement result may change due to contact between the sample and the tube wall. Loosely-supported setup has been developed in which the sample does not contact the tube wall. However, there is a gap between sample and tube wall. Also, diameter of sample is larger than inner diameter of tube. Therefore, the effect of the gap between the sample and the tube wall and the sample diameter was calculated using a finite element model. In this paper, I report the calculation results of fiber material (glass wool) using Johnson Champoux Allard (JCA) model and urethane foam using Biot-Allard model.

## 1. Introduction

Currently, when developing a sound absorbing material such as a porous body, the acoustic performance may be predicted using commercially available acoustic analysis software in order to reduce costs and shorten the development period. However, accurate material parameters (Biot parameters) are required to make predictive calculations of sound absorption using such analysis software. As the material parameter used at this time, a measured value by a dedicated measuring instrument may be used, but a value identified from the normal incident sound absorption coefficient of the impedance tube is often used. The fiber material such as urethane foam and felt having a high density may have different sound absorption coefficient measurement results depending on the contact state between the cut sample and the impedance tube wall. Naturally, the value of the Biot parameter identified therefrom also changes.

Therefore, a loose support method in which the sample and the tube wall do not contact each other has been developed and marketed (Fig. 1.). In this study, in order to investigate the effect of the gap between the sample and the tube wall, the impedance tube by the loose support method was modeled as FE and the normal incident sound absorption coefficient was calculated. The impedance tube had an inner diameter of  $\Phi 29$  mm and an analysis frequency of 500 Hz to 6300 Hz. As the porous material, glass wool (GW) and urethane foam (VO) were used, GW was JCA model, and VO was Biot model. We report on the relationship between sound absorption rate and gap.

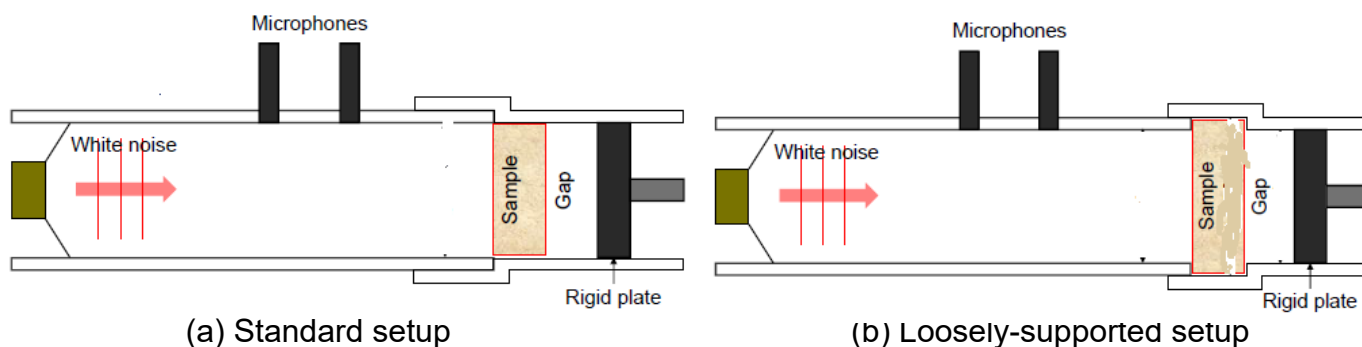


Fig. 1. Impedance tube measurement setup.

## 2. Porous material models

This time, the calculation was performed using the structural acoustic analysis software MSC. Actran. This time, the JCA model (1) was used to model glass wool. In this model, the converted density of air in the porous body is expressed by equation (1), and the converted elastic modulus is expressed by equation (2).

$$\rho_e \cong \frac{\alpha_\infty \rho_0}{\phi} \left[ 1 - j \frac{\sigma \phi}{\alpha_\infty \rho_0 \omega} \sqrt{1 + j \frac{4 \alpha_\infty^2 \mu \rho_0}{\sigma^2 \phi^2 \Lambda^2} \omega} \right] \quad (1)$$

$$K_e \cong \frac{\gamma P_0}{\gamma_0 - (\gamma_0 - 1) \left[ 1 + \frac{8 \mu}{j \Lambda'^2 B^2 \omega \rho_0} \sqrt{1 + j \rho_0 \frac{\omega B^2 \Lambda'^2}{16 \mu}} \right]^{-1}} \quad (2)$$

$\phi$  : Porosity,  $\rho_0$  : Density of air,  $\alpha_\infty$  : tortuosity,  $\sigma$  : Flow resistance,  $\omega$  : Angular frequency,  $\mu$  : Viscosity coefficient of air,  $\Lambda$  : Viscosity characteristic length,  $\gamma_0$  : Specific heat ratio of air,  $\Lambda'$  : Thermal characteristic length,  $B^2$  : Prandtl number of air.

As the urethane foam, Biot-Allard model (1) was used. This is a theoretical formula that predicts the displacement of solid-propagating sound that travels inside the material (skeleton) and air-borne sound transmitted by incident sound passing through gaps in porous elastic body. Displacement of the skeleton considering the interaction between solid-borne sound and air-borne sound :  $\vec{u}^s$  and fluid displacement :  $\vec{u}^f$  are expressed as in equations (3) and (4), respectively.

$$((1 - \phi) \rho_s + \rho_a) \frac{\partial^2 \vec{u}^s}{\partial t^2} - \rho_a \frac{\partial^2 \vec{u}^f}{\partial t^2} = (P - N) \vec{\nabla} (\vec{\nabla} \cdot \vec{u}^s) + Q \vec{\nabla} (\vec{\nabla} \cdot \vec{u}^f) + N \nabla^2 \vec{u}^s - \sigma \phi^2 G(\omega) \frac{\partial}{\partial t} (\vec{u}^s - \vec{u}^f) \quad (3)$$

$$(\phi \rho_f + \rho_a) \frac{\partial^2 \vec{u}^f}{\partial t^2} - \rho_a \frac{\partial^2 \vec{u}^s}{\partial t^2} = R \vec{\nabla} (\vec{\nabla} \cdot \vec{u}^f) + Q \vec{\nabla} (\vec{\nabla} \cdot \vec{u}^s) + \sigma \phi^2 G(\omega) \frac{\partial}{\partial t} (\vec{u}^s - \vec{u}^f) \quad (4)$$

$\phi$  : porosity,  $\rho_s$  : Density of the porous frame,  $\rho_f$  : fluid density (In this paper is air) ,  $\rho_a$  : Equivalent density of the fluid taking into consideration viscous damping in interaction of the skeleton and fluid.  $\rho_a$  is shown in equation (3).

$$\rho_a = \alpha_\infty \rho_f \left( 1 + \frac{\phi \sigma}{j \omega \rho_f \alpha_\infty} G(\omega) \right) , \quad G(\omega) = \sqrt{1 + j \frac{4 \alpha_\infty^2 \eta \rho_f \omega}{\sigma^2 \Lambda^2 \phi^2}} \quad (5)$$

$\eta$  : Solid loss coefficient,  $\sigma$  : Flow resistance,  $\alpha_\infty$  : Tortuosity factor,  $\Lambda$  : Viscosity characteristic head

Next is elastic modulus  $P$  ,  $Q$  ,  $R$  , shown in equation (6)

$$\left. \begin{aligned}
 P &= \frac{(1-\phi)\left(1-\phi-\frac{K_b}{K_s}\right)K_s + \phi\frac{K_s}{K_e}K_b}{1-\phi-\frac{K_b}{K_s} + \phi\frac{K_s}{K_e}} + \frac{4}{3}N \\
 Q &= \frac{\left(1-\phi-\frac{K_b}{K_s}\right)\phi K_s}{1-\phi-\frac{K_b}{K_s} + \phi\frac{K_b}{K_s}} \\
 R &= \frac{\phi^2 K_s}{1-\phi-\frac{K_b}{K_s} + \phi\frac{K_s}{K_e}}
 \end{aligned} \right\} \quad (6)$$

$K_b$  : Bulk modulus of skeleton(vacuum),  $N$  : Shear modulus of skeleton(vacuum),  $K_b$ ,  $N$ , shown in equation (7).

$$\left. \begin{aligned}
 N &= \frac{E(1+j\eta)}{2(1+\nu)} \\
 K_b &= \frac{2(1+\nu)}{3(1-2\nu)}N
 \end{aligned} \right\} \quad (7)$$

$P_0$  : Pressure of the equilibrium,  $\zeta$  : Thermal diffusivity.

### 3. Analysis result

#### 3.1 FE model and Biot parameters

The impedance tube FE model used in this analysis is shown in Fig. 2. Length 350 mm, and the thickness of GW was 20 mm. The mesh pitch was 0.5 mm in the radial direction.  $P_1$  and  $P_2$  are the sound pressures of the microphones 1 and 2 attached to the each positions. The transfer function was required from the sound pressures of the microphones 1 and 2, and the normal incident sound absorption coefficient was calculated. Table 1 shows the Biot parameters of the glass wool and urethane foam used in this calculation.

$$H(\omega) = \frac{P_2(\omega)}{P_1(\omega)} \quad (8)$$

$$Z_0 = \frac{P_0}{u_0} = j\rho c \frac{-H(\omega) \sin kL_x + \sin k(L_x + D_x)}{H(\omega) \cos kL_x - \cos k(L_x + D_x)} \quad (9)$$

$$\alpha_0 = 1 - \left| \frac{Z_0 - \rho c}{Z_0 + \rho c} \right|^2 \quad (10)$$

$H(\omega)$  : Transfer function,  $Z_0$  : Acoustic impedance,  $\alpha_0$  : Normal incident sound absorption coefficient,  $L_x$  : The distance of microphone 1 and cut sample,  $D_x$  : The distance of microphone 1 and 2,  $\rho$  : Air density,  $c$  : Sound velocity,  $u_0$  : Particle velocity of air,  $k$  : Wave number ( $= \frac{\omega}{c}$ )

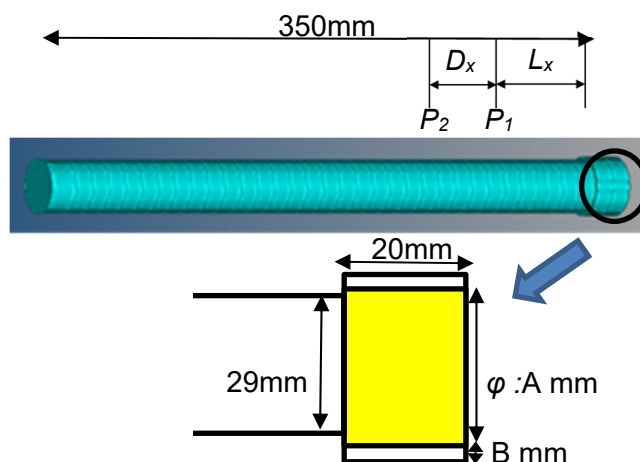


Fig. 2. Fe model of impedance tube.

Table.1 Biot parameters.

	Unit	GW	VO
Thickness	mm	20	20
Porosity	-	0.99	0.968
Flow resistivity	Ns/m <sup>4</sup>	12000	46000
Tortuosity	-	1.01	3
Viscous characteristics length	μm	130	80
Thermal characteristics length	μm	264	95
density	kg/m <sup>3</sup>	32	34
Young's modulus	Pa	-	100000
Poisson's ratio	-	-	0.39
loss factor	-	-	0.09

### 3.2 Calculation result (glass wool)

Fig. 3 shows the comparison of sound absorption rate that calculation result by the transfer matrix method [1] (TMM in the figure), calculation results of a normal  $\phi 29$ mm impedance tube FE model (FEM in the figure), calculation results when sample diameter  $A = 29$  mm and clearance  $B = 0.5$  mm ( $\phi 29\_0.5$ mm in the figure), calculation results when sample diameter  $A = 29$  mm and clearance  $B = 1.0$  mm ( $\phi 29\_1.0$ mm in the figure). From the figure, TMM and FEM are slightly different above about 4500 Hz, but the other frequencies are the same, and it can be said that the analysis accuracy of this FE model is sufficient [2] [3]. It can also be seen that the sound absorption rate tends to decrease at about 1000 Hz or more as the gap increases. In the case of loose support, it is considered that the correct sound absorption rate can be measured when the gap is as small as possible.

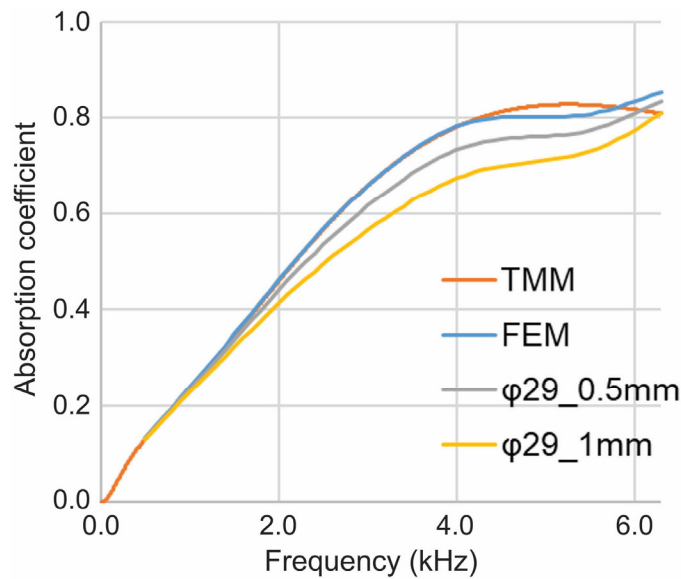


Fig. 3. Calculation results of absorption coefficient of TMM and FEM.

Fig. 4 shows the comparison of sound absorption rate that calculation result of a normal  $\phi 29\text{mm}$  impedance tube FE model (FEM in the figure), calculation results when sample diameter  $A = 29\text{ mm}$ , and clearance  $B = 1.0\text{ mm}$  ( $\phi 29\_1\text{mm}$  in Fig. 4), calculation results when sample diameter  $A = 32\text{ mm}$ , and clearance  $B = 1.0\text{ mm}$  ( $\phi 32\_1\text{mm}$  in Fig. 4). For the sample fixation, the sample diameter may be larger than the inner diameter of the impedance tube, but it is thought that the sound absorption coefficient measurement results will be affected, so if the sample diameter is the same as the inner diameter, the correct sound absorption coefficient can be measured.

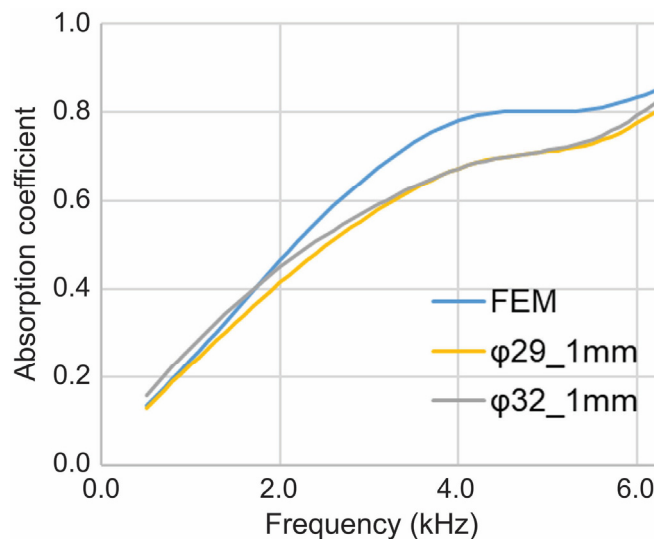


Fig. 4. Calculation results of absorption coefficient of  $\phi 29$  and  $\phi 32$ .

In Fig. 5 shows the comparison of calculation results, only the flow resistance value is multiplied by 10 ( $120000\text{ Ns} / \text{m}^4$ ), the remaining parameters are the same values as in Figs. 3 and 4 (see Table 1), and a TMM, FEM, and loose support calculation results with clearance  $B = 0.5\text{ mm}$ . TMM and FEM show a good agreement, but the sound absorption coefficient of the loosely supported model is larger than before. A certain flow resistance value agrees with TMM, and if it is smaller than that, the sound absorption coefficient is small, and if it is larger, the sound absorption coefficient is large.

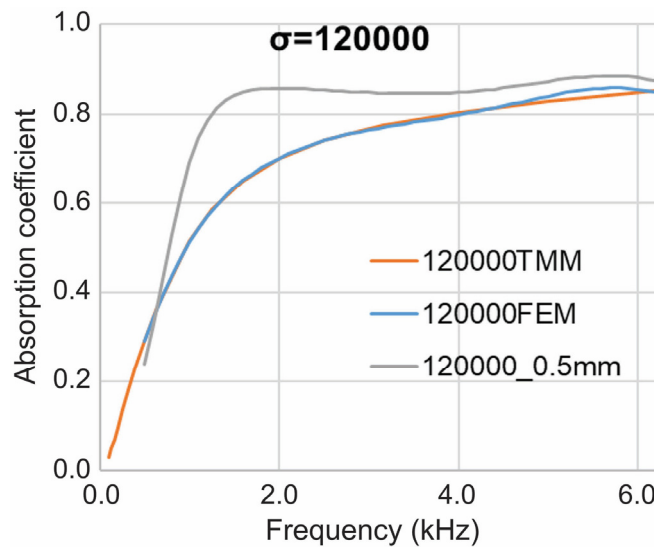


Fig. 5. Calculation results of absorption coefficient of TMM and FEM (Flow resistivity $\times$ 10).

Fig. 6 shows the comparison of the sound absorption coefficient of VO calculated using the Biot model. With this parameter, the calculation result of the loose support method was slightly smaller.

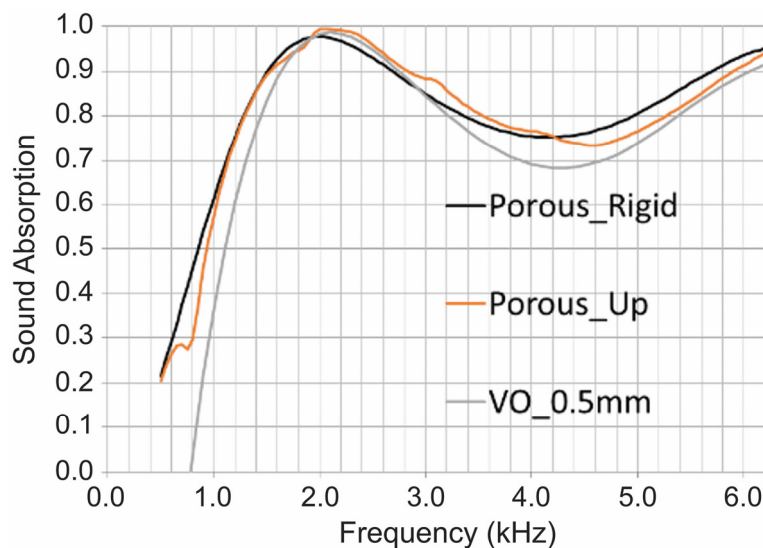


Fig. 6. Calculation results of absorption coefficient of TMM and FEM (VO).

#### 4. Conclusion

The effect of the gap of the loose support method of impedance tube measurement was calculated using the FE model, and the following results were obtained.

- The gap should be as small as possible.
- The sample diameter should be the same as the inner diameter of the pipe as much as possible.

In the future, further parameter studies will be conducted.

References

- [1] K. Hirose, H. Suzuki, H. Nakagawa, and K. Takahashi, "Parameter study on Biot model of porous elastic material at normal incident sound absorption coefficient", *Japan Acoustical Society Building Acoustics Study Group*, April 28, 2015.
- [2] J. F. Allard, and N. Atalla, "Propagation of sound in porous media", *John Wiley & Sons, Inc.*, 2009.
- [3] U. Hayase, Y. Kurosawa, "Influence of sample boundary conditions on impedance tube measurement", *Society for Vibration Control Engineering 30th anniversary technology exchange meeting*, SDT17016, 2017.
- [4] Y. Kurosawa, "Calculation of normal incidence sound absorption coefficient of sound absorbing material for automobiles using FE model of impedance tube", *Dynamics and Design Conference 2018*, No.324, 2018.
- [5] Y. Kurosawa, "Analysis of the influence of the gap of the loose support method in impedance tube measurement", *Acoustical Society of Japan 2018 Autumn research presentation*, 2018.