# Finite Element Analysis for Damping Properties of a System Having Liquid, Elastic Body and Viscoelastic Body

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**Abstract.** An analytical method is proposed to calculate damping properties for a system involving liquid, elastic body and viscoelastic body in a three-dimensional region. Particle velocity is selected as an unknown parameter for the proposed method. Further, damping effects of viscoelastic body on the system are analyzed. Water, a steel water tank and damping layers were used for analysis as liquid, elastic body and viscoelastic body. When damping layers were attached on tank's wall, modal loss factors increase for coupled modes between elastic deformation of the tank's walls and resonance of the water.

## 1. Introduction

Many companies use CAE in the product development process to significantly reduce development speed and costs. Therefore, although there are many commercially available CAE softwares, they cannot often use appropriate values for the dissipation term during design. This is because damping elements that cause dissipative effects into the calculation, lead to complicated calculation method and they require an enormous amount of calculation time. Yamaguchi [1]-[3]. proposed a method that a high-speed calculation of large-scale degree-of-freedom problems for mixtures of elastic bodies, viscoelastic bodies, porous bodies, gases and liquids under arbitrary shapes by improving conventional finite element methods. In this report, we extend this method and propose a damping characteristic analysis method using the finite element method for a system consisting of a liquid, an elastic body, and a viscoelastic body with the particle displacement as an unknown variable.

## 2. Analytical Model and Boundary Conditions

An analytical model is shown in Figure 1. To understand behaviors of this coupled system among liquid, elastic body, and viscoelastic body, a steel structure (cuboid shaped water tank) with bottom ($x$, $y$) = (40cm, 30cm), depth $z$ = 40 cm, and thickness 2 mm is used. In this tank, a liquid (water) is filled to a depth of 20cm. We call this model as Case 1. To investigate effectiveness of the damping effect, we analyze a model in which damping material with a thickness of 2.5 mm is laminated. In this case, only on the side surface in the x direction, we set the damping material. We call this model as Case 2. In this analysis, we assume that the liquid has sufficiently low viscosity. In addition, we neglect friction. And there is no sloshing at the water surface, the pressure was kept constant at atmospheric pressure as a boundary condition for the liquid surface. The sound source position is the center of the water, and the sound source waveform is white noise. The observation point is taken outside the tank wall. Figure 2 shows a cross-sectional view of the model at $y$ = 15cm. Elements are created by dividing water into 20 parts in the $x$ direction, 16 parts in the y direction, and 20 parts in the z direction, and the water tank and damping material were also divided into element sizes similar to the water.

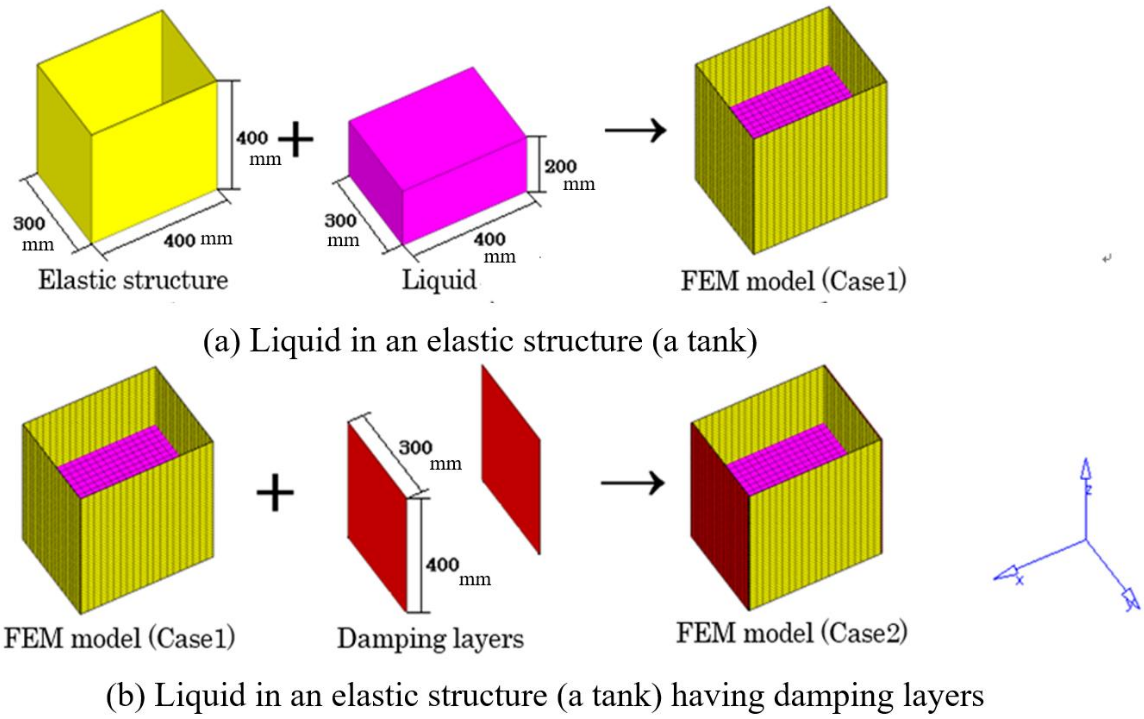



Fig.1. Analytical model of liquid in a tank having damping layers.

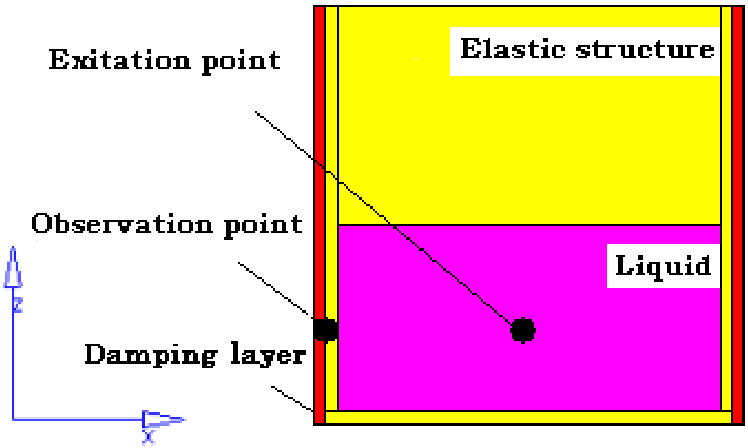



Fig. 2. Cross section of analytical model for liquid in a tank having damping layers.

## 3. Numerical Method [1]-[3]

A three-dimensional sound field in a micro viscous liquid is discretized using finite elements. The equation of motion of a periodically excited inviscid compressible perfect fluid under the conditions of neglecting gravity and small amplitude is as follows.

$$\nabla \ s = -\rho\omega^2 \{u_f\} \tag{1}$$

Since we are considering the propagation of sound waves in liquid, taking compressibility into account, the relationship between pressure and volumetric strain is as follows,

$$s = E\left\{\frac{\partial u_{fx}}{\partial x} + \frac{\partial u_{fy}}{\partial y} + \frac{\partial u_{fz}}{\partial z}\right\} \quad , \quad s = -p \tag{2}$$

Where, $p$ : sound pressure, $E$ : bulk modulus of elasticity, $\rho$ : mass density, $\omega$ : angular frequency, particle displacement $\{u_f\} = \{u_{fx}, u_{fy}, u_{fz}\}^{\mathrm{T}}$ , $u_{fx}, u_{fy}, u_{fz}$ :components of particle displacement in the $x$, $y$ and $z$ directions. Considering the non-rotating condition, we obtain the element equation of motion from the Lagrangian equation.

$$\left([K]_{fe} - \omega^2 [M]_{fe}\right)\{u_{fe}\} = \{f_{fe}\} \tag{3}$$

$$[M]_{fe} = \rho_e [\tilde{M}]_{fe} \tag{4}$$

$$[K]_{fe} = E_e [\tilde{K}]_{fe} \tag{5}$$

Where, $\{f_{fe}\}$ is the nodal displacement vector of the liquid element, $[K]_{fe}$ is the element stiffness matrix and $[M]_{fe}$ is the element mass matrix. $[\tilde{K}]_{fe}$ and $[\tilde{M}]_{fe}$ show the matrix including the shape functions and their derivatives.

To consider Modeling dissipation in micro viscous liquids by introducing complex density [5] and complex bulk modulus [2], [3].

$$\rho_e \Rightarrow \rho_e^* = \rho_{\mathrm{eR}} + j\ \rho_{\mathrm{eI}} \tag{6}$$

$$E_e \Rightarrow E_e^* = E_{\mathrm{eR}} + j\ E_{\mathrm{eI}} \tag{7}$$

Where, $\rho_{\mathrm{eR}}$ :the real part of $\rho_e^*$, $\rho_{\mathrm{eI}}\left(=-\frac{R}{\omega}\right)$ :the imaginary part of $\rho_e^*$, $R$ : flow resistance, $E_{\mathrm{eR}}$ ::the real part of $E_e^*$, $E_{\mathrm{eI}}$ : the imaginary part of $E_e^*$, $j$ :imaginary unit

Substituting equation (6) into equation (4) yields the following equation.

$$[M]_{fe} = [M_{\mathrm{R}}]_{fe}\left(1 + j\ \chi_e\right) \ , \ \chi_e = \rho_{\mathrm{eI}} / \rho_{\mathrm{eR}} \tag{8}$$

Substituting equation (7) into equation (5) yields the following equation.

$$[K]_{fe} = [K_{\mathrm{R}}]_{fe}\left(1 + j\ \eta_e\right) \ , \ \eta_e = E_{\mathrm{eI}} / E_{\mathrm{eR}} \tag{9}$$

Where, $[M_{\mathrm{R}}]_{fe}$: the real part of $[M]_{fe}$, $[K_{\mathrm{R}}]_{fe}$: the real part of $[K]_{fe}$, $\chi_e$ : material damping due to flow resistance, : material damping due to bulk modulus of liquid

Above, we can express the energy dissipation in the elements for a micro viscous liquid using complex numbers.

When the vibration field of a solid is discretized using linear finite elements, it becomes the following equation.

$$\left([K]_{se} - \omega^2 [M]_{se}\right)\{u_{se}\} = \{f_{se}\} \tag{10}$$

$$[K]_{se} = [K_{\mathrm{R}}]_{se}\left(1 + j\,\eta_e\right) \tag{11}$$

Where, $\{f_{se}\}$: nodal force vector for solid elements, $[M]_{se}$ : element mass matrix, $\eta_e$ : material loss factor of an element $e$, $[K]_{se}$ : element stiffness matrix, $[K_{\mathrm{R}}]_{se}$ : the real part of $[K]_{se}$.

At the boundary between solid and liquid, only the displacement normal to the boundary is continuous. When all the elements (liquid, solid) are superimposed and the entire system discretized equation is obtained, as follows.

$$\sum_{e=1}^{e\max}\left([K_{\mathrm{R}}]_e(1 + j\eta_e) - \omega^2 [M_{\mathrm{R}}]_e (1 + j\chi_e)\right)\{u_e\} = \{f\} \tag{12}$$

Where, $\{f\}$: nodal force vector, $\{u_e\}$: nodal displacement vector. $\{u_e\}$ is consisted of $\{u_{fe}\}$ and $\{u_{se}\}$. $[K_{\mathrm{R}}]_e$ is comprised of $[K_{\mathrm{R}}]_{fe}$ and $[K_{\mathrm{R}}]_{se}$. $[M_{\mathrm{R}}]_e$ is consisted of $[M_{\mathrm{R}}]_{fe}$ and $[M_{\mathrm{R}}]_{se}$.

Next, we consider the approximate calculation of the modal damping of the entire system. The complex eigenvalue problem of equation (12) is as follows.

$$\sum_{e=1}^{e\max}\left([K_{\mathrm{R}}]_e(1 + j\eta_e) - \left(\omega^{(n)}\right)^2\left(1 + j\eta_{tot}^{(n)}\right)[M_{\mathrm{R}}]_e(1 + j\chi_e)\right)\left\{\phi^{(n)^*}\right\} = \{0\} \tag{13}$$

Next. we introduce an infinitesimal quantity and asymptotically expanding the solution of equation (13). By summarizing the infinitesimal quantities up to the first-order terms, we obtain the following approximate expression for the modal loss factor.

$$\eta_{tot}^{(n)} = \eta_{se}^{(n)} - \eta_{ke}^{(n)}, \eta_{se}^{(n)} = \sum_{e=1}^{e\max}\left(\eta_e S_{se}^{(n)}\right), \eta_{ke}^{(n)} = \sum_{e=1}^{e\max}\left(\chi_e S_{ke}^{(n)}\right) \tag{14}$$

Where, $\eta_{tot}^{(n)}$:modal loss factor, $S_{se}^{(n)}$:share of strain energy of an element $e$ to entire system, $S_{ke}^{(n)}$: share of kinetic energy of an element $e$ to entire system.

The relative amount of dissipated energy shared by each element is given by the following equation.

$$D_e^{(n)} = \eta_e S_{se}^{(n)} - \chi_e S_{ke}^{(n)} \tag{15}$$

Using these values, we compute the frequency responses using the modal method. Hereafter, eight-nodes isoparametric hexahedral elements are used as the finite elements corresponding to the liquid. The real part of the bulk modulus of elasticity $E_{\mathrm{eR}}$ is $2.22 \times 10^9$ N/m$^2$, the real part of the mass density $\rho_{\mathrm{eR}}$ is $1.00 \times 10^3$ kg/m$^3$ and the material loss factors $\eta_e$ and $\chi_e$ for the liquid are 0.001 and -0.001, respectively. For the solid elements, we used eight-nodes isoparametric hexahedral elements that took into account nonconforming modes [6]. For the steel elements, the Young's modulus is $210 \times 10^9$ N/m$^2$, mass density is $7.80 \times 10^3$ kg/m$^3$, material loss factor $\eta_e$ is 0.001. For the viscoelastic damping material, the storage modulus elasticity is $8.00 \times 10^8$ N/m$^2$, mass density is $1.45 \times 10^3$ kg/m$^3$ and the material loss factor is 1.00, respectively.

## 4. Analysis results and considerations

In this report, we proposed a damping analysis method using the finite element method for a system consisting of a liquid, an elastic body, and a viscoelastic body, where the particle displacement is as unknown variable.

First, we compared the frequency responses of Case 1 (without damping material) and Case 2 (with damping material) as shown in Figure 3. The peak frequencies are lowered by laminating the damping material. This is because the increase rate of the mass due to the lamination of the damping material is greater than the increase rate of the stiffness. The peak level at the resonance points are smaller in Case 2 than in Case 1. This shows that layering damping materials is effective as a general damping method for this type of systems [4].

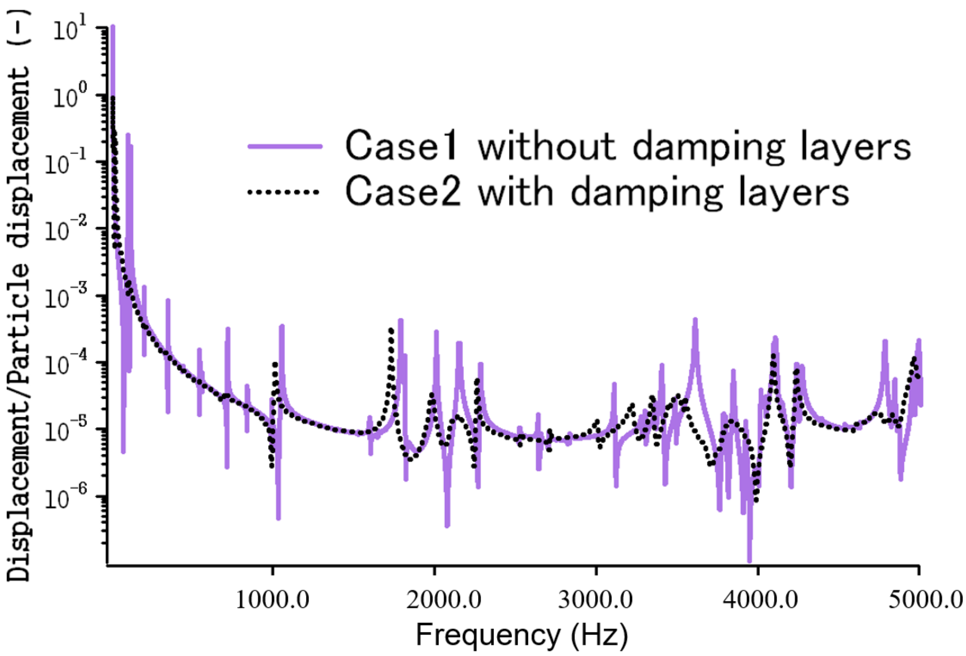
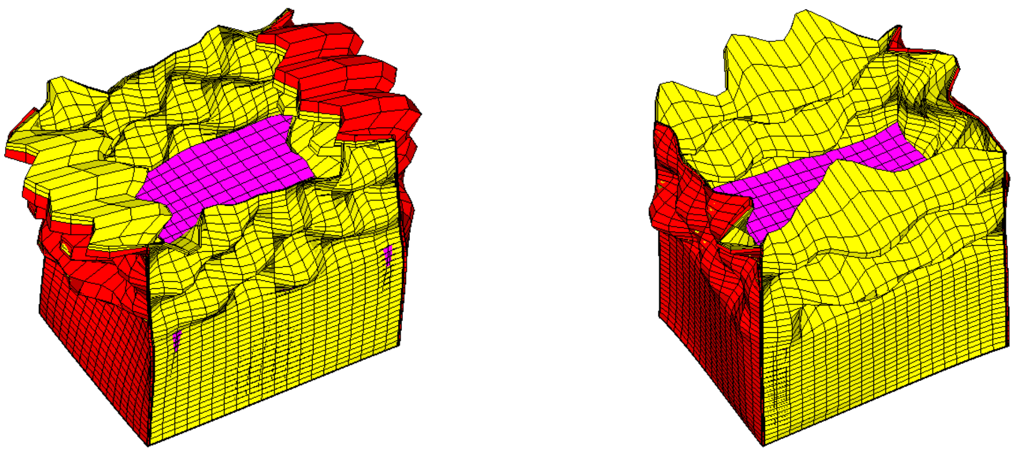



Fig. 3. Frequency responses of liquid in a tank with / without damping layer.

Table 1 shows a comparison of the analytical values and experimental values [4] of the modal loss factors and resonant frequencies. Comparing the modal loss factors with and without damping material, the analytical values with the damping material are approximately 15 times higher than those without the damping material. As a result, we can obtain high damping effects. And the experimental modal loss factors [4] with the damping material are approximately 10 times higher than those without the damping material. Qualitatively, the experimental values and the analytical values of the loss factors are consistent. In the actual experiment, it is assumed that a higher modal loss factors than in the analysis were measured due to friction at the contact parts when installing the experimental equipment. And friction at the joints between the walls of the aquarium are also added. The corrected value of the loss factor caused by this friction is set to $\varDelta\eta$ = 0.0015. This was determined from the difference in the modal loss factor between the experimental value and the analytical value without damping material. When $\varDelta\eta$ is added to the analytical value with damping material, the analytical corrected value is approximately 0.02, which is almost the same as the experimental value. Consequently, the corrected loss factor agree well quantitatively.

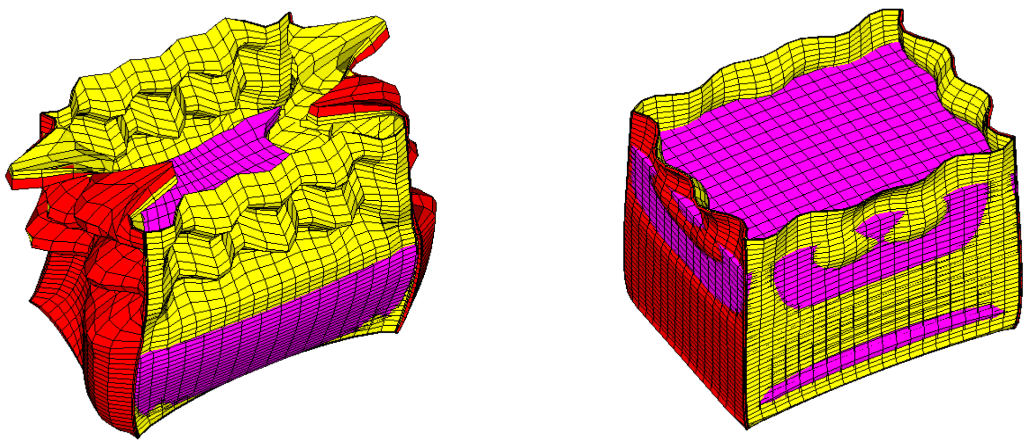
Table.1. Comparison between analytical values and experimental values [4].

| | | With damping material | Without damping material |
|---|---|---|---|
| Analytical value | Modal loss factor (-) | 0.018 | 0.0012 |
| | Resonant frequency (kHz) | 4.76 | 4.79 |
| Experimental value [4] | Modal loss factor | 0.024 | 0.0027 |
| | Resonant frequency (kHz) | 4.44 | 4.56 |
| Correction value | Modal loss factor (-) | 0.020 | 0.0027 |



(a) Resonance at 3.50 kHz (b) Resonance at 3.01 kHz

Fig. 4. Typical examples of bending modes of tank’s wall.



(c) Resonance at 3.35 kHz (d) Resonance at 1.73 kHz

Fig. 5. Typical examples of eigenmodes in water.

The vibration modes obtained by analysis are classified into two types. One type is mainly caused by bending deformation of the tank walls. The other type is mainly caused by the resonance of the water in the tank. Figures 4 and 5 show examples of the main mode of bending deformation of the tank wall and the main mode of resonance of water that appeared in the analysis.

Table 2 shows the modal loss factors and resonance frequencies with and without damping material for each mode (a) to (d) shown in Figs. 4 and 5. In the modes (a) and (b) where the bending deformation of the tank wall are dominant, the modal loss factors of the mode (a), where the *x*-direction deformation is the main component, is more than 10 times that of the case without damping material. On the other hand, the modal loss factors of the mode (b), which mainly has the *y*-direction deformation, is about five times that of the case without damping material. From this, it can be seen that the higher damping effect can be obtained, when the larger deformations appear in the tank wall with the damping material.

Next, we investigate changes in the modal loss factors for modes (c) and (d) having large deformations in the water. Although it is not as large as damping of modes having large bending deformations in the tank wall, the modes for large deformations in the water also have damping effects. The modal loss factor of mode (c), which mainly consists of deformations in the *x*-direction, is approximately three times as large than those without the damping material. The modal loss factor of mode (d), which mainly consisted of deformations in the *z*-direction are approximately 1.5 times than those without the damping material. It can be seen that even in the coupled modes where water's resonances are dominant, the vibration modes in which the larger deformations of the tank walls layered with the damping materials, obtain the higher vibration damping effects. The reason why the resonance-dominant modes of the water also has a damping effect is that the damping material laminated on the tank wall attenuates the vibration of the liquid part in contact with the tank walls. The damping material that is not in direct contact with the water, contributes to the attenuation of the modes relating the resonances in the water.

Table. 2. Resonant frequencies and modal loss factors (a)～(d).

| | | With damping layers | Without damping layers |
|---|---|---|---|
| (a) Resonance at 3.50 kHz | Modal loss factor (-) | 0.021 | 0.0017 |
| | Resonant frequency (kHz) | 3.50 | 3.61 |
| (b) Resonance at 3.01 kHz | Modal loss factor (-) | 0.0055 | 0.0012 |
| | Resonant frequency (kHz) | 3.01 | 3.11 |
| (c) Resonance at 3.35 kHz | Modal loss factor (-) | 0.0047 | 0.0018 |
| | Resonant frequency (kHz) | 3.35 | 3.41 |
| (d) Resonance at 1.73 kHz | Modal loss factor (-) | 0.0020 | 0.0013 |
| | Resonant frequency (kHz) | 1.73 | 1.79 |

## 5. Conclusion

(1) For a system in which a liquid, an elastic body, and a viscoelastic body are coupled, a damping characteristic analysis method using the finite element method was proposed by introducing particle displacement as unknown variable. And its validity was confirmed.

(2) By layering damping materials on the tank walls, we can increase damping not only for modes including elastic deformations in tank walls, but also for modes including resonances of liquid.

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