

Numerical Computation of Urethane Foam's Nonlinear Compression Response through Microstructural Deformation Analysis

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Keywords: finite element method, nonlinear restoring force, Kelvin cell, buckling

Abstract. When compressed, urethane foam exhibits a soften-hardening behavior in terms of its resilience. Notably, when compressed up to 33%, the stiffness of the foam significantly decreases. To investigate the underlying mechanism of this resilience behavior under compression, a microstructural model was employed. The microcellular structure of the urethane was modeled using a Kelvin cell geometry and implemented as a finite element model. Numerical deformation analysis of the Kelvin cell model was conducted using LS-Dyna, a nonlinear finite element method code. The study revealed that the softening mechanism in the foam's resilience arises from the geometric nonlinearity of the resin skeleton constituting the Kelvin cell structure.

1. Introduction

Various soundproofing materials have been applied to ensure acoustic comfort within automobile cabins. These materials are often composed of porous media, such as foam or fibrous materials, which are installed on vehicle body panels. In some cases, a hard cover is applied to the surface of the porous material, forming a double-wall structure that functions as a sound-insulating system [1]-[7].

The following research has been conducted on the problem of foam cell deformation.

Beginning with the fundamental research of Gibson and Ashby [8], the problem of foam deformation under compressive load has been studied, and it has been shown that softening occurs due to geometric nonlinearity and buckling of the cell struts. Gibson and Ashby considered the Euler buckling of a simple cubic model and obtained an analytical solution for the buckling load of the foam. By Laroussi et al., similar conclusions have also been obtained using periodic microstructure models [9].

Gong et al. [10] analyzed the microscopic buckling of Kelvin foam using the finite element method and found long-wavelength buckling modes that depend on the periodic unit length.

By Jang et al. [11], two families of finite element models of anisotropic, aluminum alloy, open-cell foams are developed and their predictions of elastic properties and compressive strength are evaluated by direct comparison to experimental results. In the first family of models, the foams are idealized as anisotropic Kelvin cells loaded in the <100> direction. And the second family more realistic models, based on Surface Evolver simulations of random soap froth with N3 cells, are constructed.

Calati et al. investigated the orientation and forced convection of a Kelvin cell porous structure in air [12].

By Carlsson et al. [13], periodic unit-cell solutions are obtained for the finite-strain, elasto-plastic response of a filled closed-cell Kelvin foam in uniaxial compression.

These studies deal with deformation problems involving buckling under static loads, and do not involve dynamic loads.

Meanwhile, in the case of vibro-acoustic problems, Yamamoto et al. used the results of finite element analysis of the coupling between acoustics and structure within the cells of porous materials to determine bio parameters and sound absorption characteristics using the homogenization method [14].

However, no studies have been conducted on polyurethane foam used in automotive soundproofing under compressed conditions.

In this report, we use the Kelvin cell model to investigate the softening and buckling behavior of compressed polyurethane foam cells in sound-absorbing double walls and their affections on sound insulation performance.

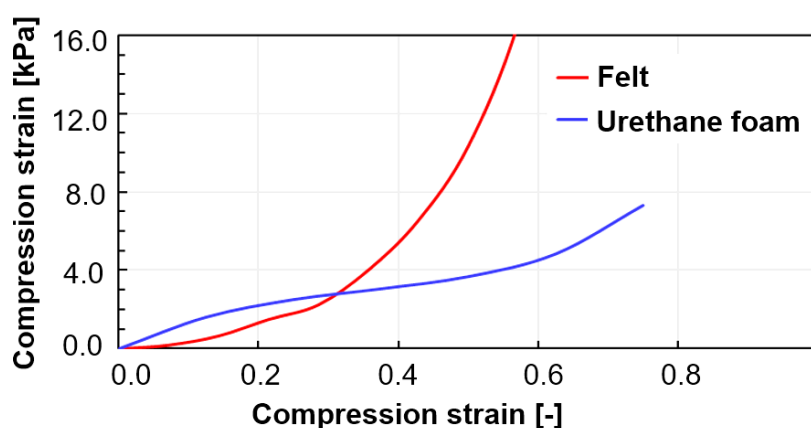


Fig. 1. Experimental results for stress-strain curve of Urethane foam (blue) and felt (red) under compression.

When the porous material located between the double-wall structure is urethane foam, it has been demonstrated that compressing the foam enhances sound insulation performance, particularly at a compression ratio of 33% [1]. As shown in Fig.1, this improvement is attributed to the unique compression characteristics of urethane foam, which differ from those of felt and exhibit a gradually softening and hardening restoring force behavior. It has been concluded that the decrease in linear elastic Young's modulus at 33% compression enhances the acoustic insulation capability of the double-wall system.

In this study, to investigate the origin of the gradually softening and hardening restoring force behavior of urethane foam, the foam's microcellular structure was modeled using Kelvin cells. A nonlinear dynamic deformation analysis was conducted using the finite element method (FEM). By examining the deformation behavior of the cells, the underlying mechanism responsible for the foam's nonlinear restoring force characteristics was elucidated.

2. Analysis Models and analysis method

The microcellular architecture of polyurethane foam was characterized through scanning electron microscopy (SEM) at a magnification of 120 times, as presented in Fig.2. The SEM analysis revealed that each cell exhibits a polyhedral configuration, with faces composed of hexagonal openings. Notably, no membrane structures were observed surrounding these openings. The internal framework of each cell can be interpreted as an assembly of multiple structural members, effectively modeled as resin skeleton. These members possess cross-sectional profiles resembling triangular geometries. The measured cell diameters range from approximately 345 μm to 520 μm .

Based on these morphological observations, a Kelvin-type geometric model was constructed, as illustrated in Fig.2. We selected the Kelvin cell model because the shapes of the actual cells in the SEM images were similar. The Kelvin model, widely recognized for its space-filling efficiency, incorporates both hexagonal and square apertures. A single unit cell comprises 17,104 solid elements and 6,196 nodes. When we used beams or shells for the struts, it is difficult to estimate the rigidity of the connections between the struts. Thus, we selected the solid elements for the struts. As shown in Fig.2, the model allows for seamless tiling of multiple unit cells without interstitial gaps, enabling large-scale structural representation.

The frame members within the Kelvin cells were modeled using polyurethane resin, characterized by a linear elastic Young's modulus of 0.1 [GPa] and a mass density of 1.02×10^3 [kg/m³]. The simulation framework neglects material nonlinearity and considers only geometric nonlinearity to capture deformation behavior under loading. The thickness and triangular cross-sectional geometry of the struts were determined based on SEM observations of the physical samples. To accurately capture the buckling behavior within a reasonable computational time, the element size was set to ensure at least four elements along each side of the triangular cross-section of the resin skeleton.

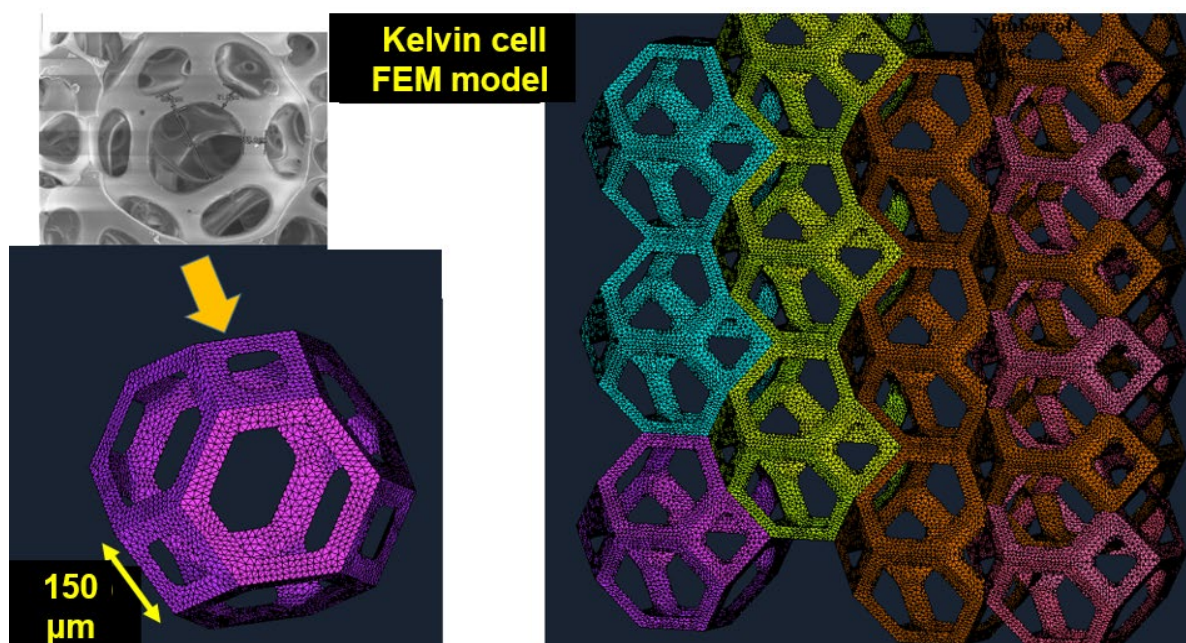


Fig. 2. Photo from SEM and FEM models for dynamic deformation analysis using microscopic cells in compressed Urethane foam.

Dynamic deformation analyses were conducted for both a single unit cell and an array of connected cells. The simulations were performed using the LS-DYNA finite element solver. A distributed load was applied to the upper surface of the cellular structure via a rigid body, and the resulting dynamic response was evaluated.

3. Analysis Results and Discussion

3.1 Analysis of the Compressive Behavior of a Single Kelvin Unit Cell

Initial simulations were conducted to evaluate the compressive behavior of a single Kelvin unit cell. Subsequently, the response of multiple connected unit cells was examined to determine whether their mechanical characteristics are consistent with those observed in the single-cell configuration.

The left graph of Fig.3 illustrates the computational model of a Kelvin unit cell. In this setup, the bottom surface of the cell is constrained, while a distributed compressive load is applied in the negative Z-direction to the top surface through a rigid block. The load was incrementally increased by 0.004 N at intervals of 0.1 seconds. The right graph of Fig.3 shows the deformation before and after compression, where dynamic buckling was observed.

Figure 4 shows the Z-directional displacement as a function of compressive load during compression. As the load increases, the slope of the displacement curve (representing the effective spring constant) gradually decreases, indicating a softening behavior in the restoring force. As the compressive stress increases, the macroscopic linear elastic Young's modulus of the unit cell decreases. With further loading, dynamic buckling accompanied by transient motion occurs, followed by a hardening response in the restoring force. Overall, the restoring force exhibits a softening-hardening transition, a behavior analogous to that observed in shell structures.

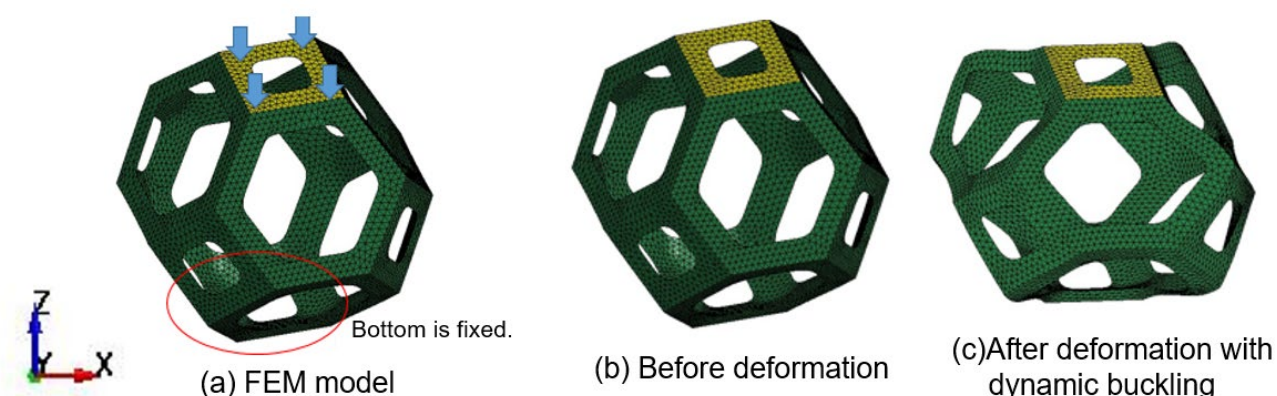


Fig. 3. A FEM model and a deformation with buckling from dynamic deformation

Subsequently, an alternative boundary condition was investigated using the same model. In this case, the bottom surface of the cell was fixed in the Z-direction but remained free in the X and Y directions. Figure 5 depicts the deformation before and after compression, revealing that the cell undergoes tilting and dynamic buckling. Figure 6 shows the Z-directional displacement as a function of compressive load during compression. Similarly to the previous condition, the slope of the displacement curve decreases over load, indicating a softening behavior in the restoring force. The macroscopic linear elastic Young's modulus of the unit cell also decreases under this condition as compressive stress increases. With further loading, dynamic buckling accompanied by transient tilting motion is observed, followed by a hardening response in the restoring force. Thus, the softening-hardening transition is consistently observed under both boundary conditions.

These results demonstrate that the softening-hardening behavior of the restoring force is a fundamental characteristic of the Kelvin unit cell, and it is therefore expected to persist in structures composed of multiple Kelvin cells.

The FEM model in this study represents a localized portion of the urethane foam microstructure; thus, the absolute values of displacement and load do not directly coincide with experimental data due to the difference in scale. Nevertheless, the simulation successfully captures the softening-hardening restoring force characteristics observed in the experiments, demonstrating qualitative agreement. Nevertheless, the compressive load–displacement relationship in Figure 6 shows that dynamic buckling occurs at a displacement of approximately 0.15 mm, leading to a significant reduction in stiffness. Since the initial cell size is 0.424 mm, the displacement at the onset of dynamic buckling corresponds to a strain of 35%, which demonstrates quantitative agreement with the experimental value of 33%."

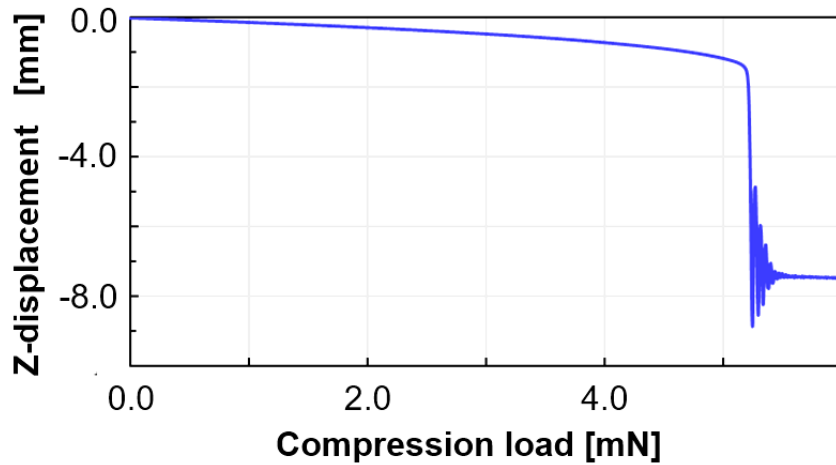


Fig. 4. Compression load and Z-displacement under compression from dynamic deformation analysis using one unit cell in Urethane foam.

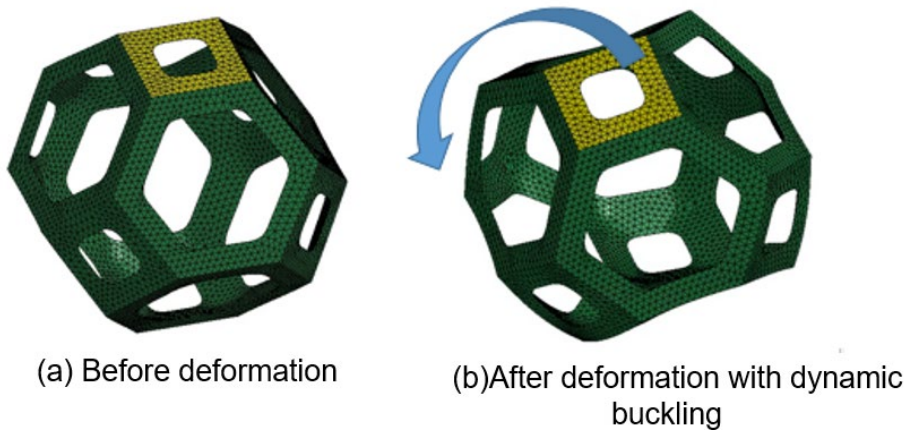


Fig. 5. Deformation with tilt and buckling from dynamic deformation analysis using one unit cell in compressed Urethane foam.

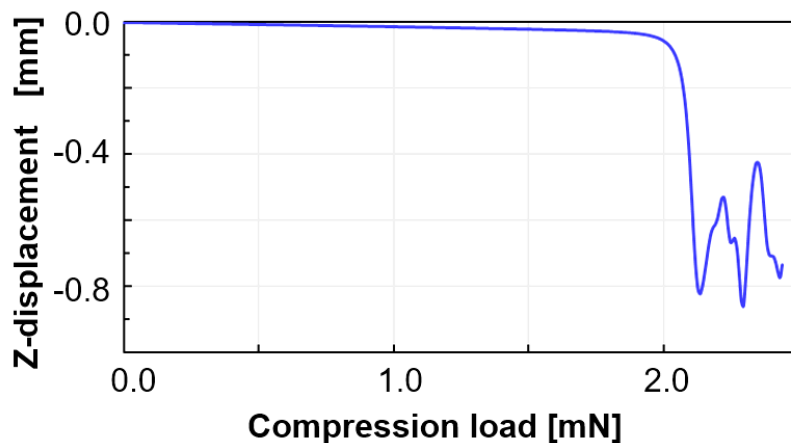


Fig. 6. Compression load and Z-displacement under compression from dynamic deformation analysis using one unit cell in Urethane foam.

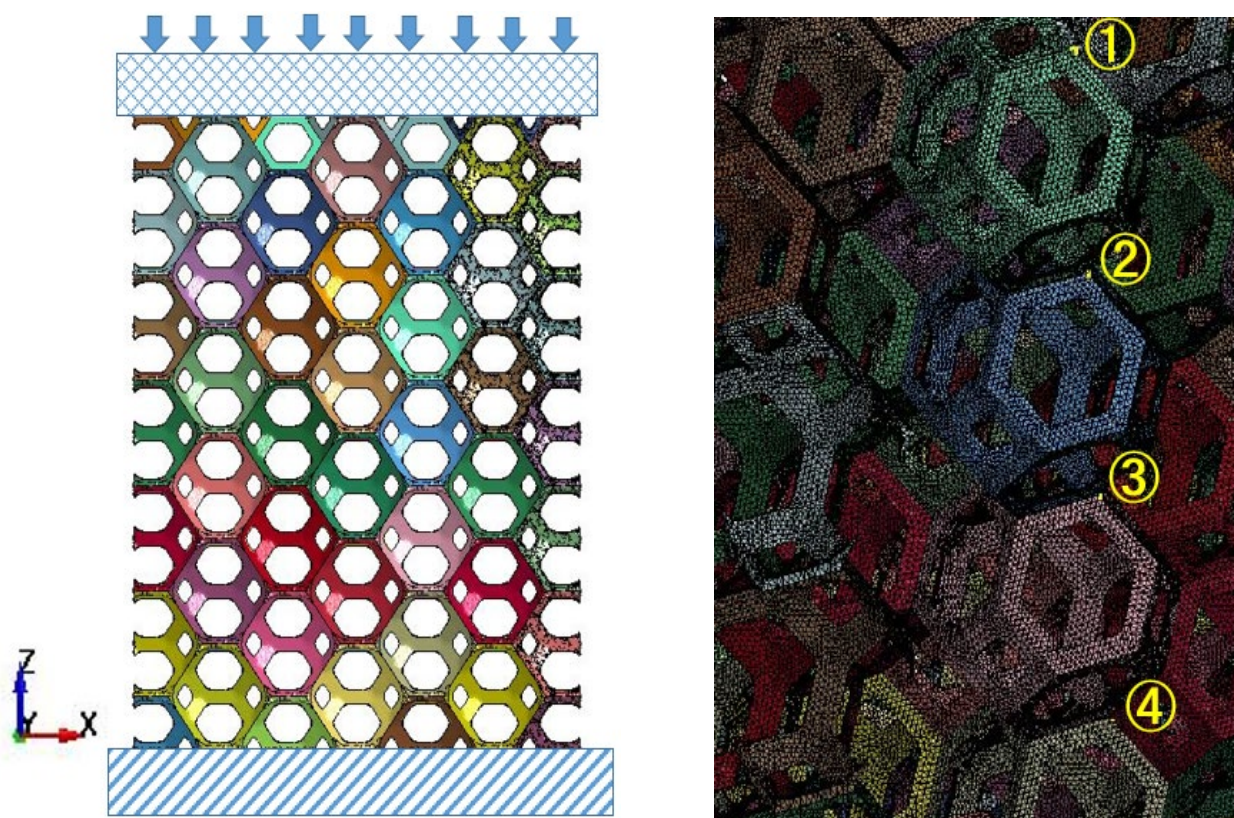


Fig. 7. FEM Kelvin model and its observation 4 points for dynamic deformation analysis using microscopic multiple cells in compressed Urethane foam.

3.2 Analysis of the Compressive Behavior of Multiple Kelvin Unit Cells

The left graph in Fig.7 illustrates the computational model used for nonlinear dynamic deformation analysis under compression, employing multiple Kelvin unit cells. The model consists of six layers of cells in the Z-direction and seven layers in both the X- and Y-directions. A distributed compressive load is applied in the negative Z-direction via a rigid block positioned at the top of the model. The load is incrementally increased by 50 N at intervals of 0.001 seconds for each node on the upper surface. We would like to clarify that the analysis presented in this study is inherently dynamic rather than quasi-static. This approach was specifically chosen to accurately capture the response of urethane foam as a soundproofing material under high-frequency conditions. For instance, at a representative evaluation point in our simulation, a displacement of 0.15 mm occurs within 0.02×10^{-3} s, resulting in an average deformation velocity of 7500 mm/s. This value is highly consistent with the deformation velocity observed in actual acoustic environments; for comparison, the maximum velocity of a 1000 Hz vibration with a 1 mm amplitude is approximately 6,283 mm/s. Since the deformation velocity in our analysis matches the order of magnitude expected for high-frequency soundproofing performance, we believe that employing a dynamic framework is both appropriate and essential for this study.

This model comprises 3,917,440 solid elements and 1,353,876 nodes, with 33,095 nodes located at the top surface where the load is applied. The simulation is conducted using LS-DYNA. The right graph in Fig.7 indicates four evaluation points placed at the top of each cell layer along the Z-direction.

Figure 8 presents the simulation results, where the vertical axis represents displacement in the Z-direction and the horizontal axis denotes load, corresponding to the applied compressive load. It is shown that, as the applied compressive load increases, the slope of the curve (corresponding to the spring constant) decreases, indicating a softening behavior in the restoring force. With increasing compressive stress, the linear elastic Young's modulus declines. It should be noted that deformation

in the lower cell layers appears delayed, which is attributed to give an intentionally fast deformation to reduce computational time.

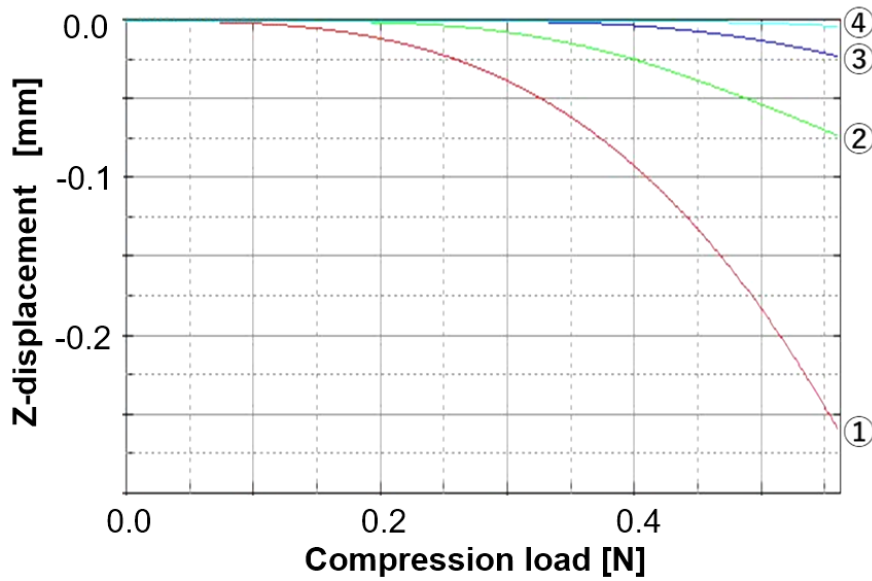


Fig. 8. Compression load and Z-displacement under compression for dynamic deformation analysis using microscopic multiple Kelvin cells in Urethane foam.



Fig. 9. Nonlinear elastic deformation before / after compression of microscopic multiple Kelvin cells in Urethane foam.

Figure 9 depicts the nonlinear elastic deformation before and after compression. As the compressive load increases, the structural members (beams in the resin skeleton) within the cell begin to buckle, exhibiting the softening-type nonlinear spring behavior described in the previous section. Consequently, the slope of the stress-strain curve (i.e., elastic modulus) decreases with increasing stress. In detail, as the compressive load increases and deformation progresses in the Z-direction, the beams gradually tilt, resulting in a reduction of axial deformation (associated with high stiffness) and an increase in bending deformation (associated with low stiffness). This leads to an overall softening of the cell structure, which may eventually result in buckling due to geometric nonlinearity.

Therefore, the primary factor contributing to the softening of urethane foam after 33% compression is attributed to geometric nonlinearity.

It is noted that in the multiple Kelvin cell model, adjacent nodes at the boundaries between single Kelvin cell units were rigidly constrained to ensure that their almost displacements coincided.

These results of this study suggest that the softening restoring force characteristics are primarily governed by dynamic buckling involving bending deformation of the resin skeleton. Considering this mechanism, the sensitivity to skeleton thickness is interpretable within the conventional framework of general buckling problems. We would like to study sensitivities to cell size/strut thickness/boundary conditions (periodic vs. fixed) in our future works.

Figure 10 shows the experimental setup for insertion loss measurement. Sound is emitted from a speaker inside a reverberation box, and the sound pressure level is measured by a microphone positioned above. The difference in sound pressure levels is evaluated as the insertion loss.

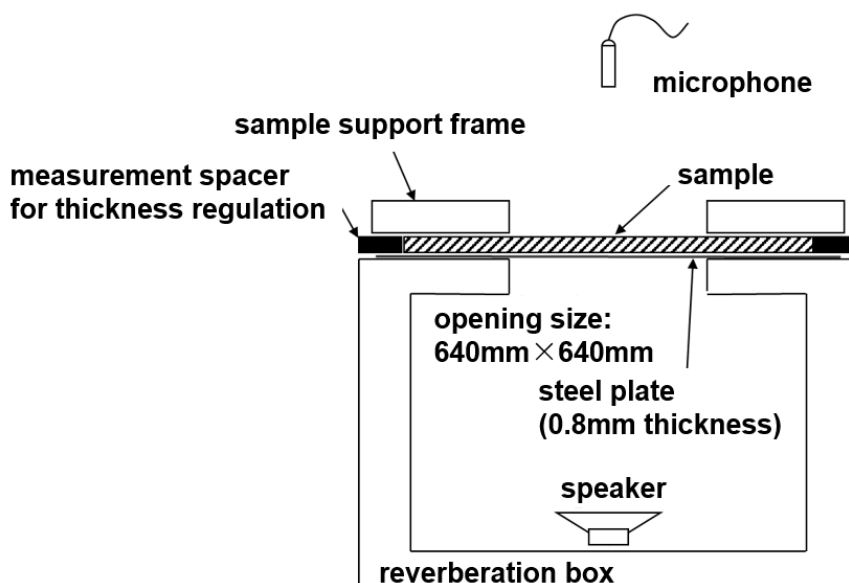


Fig. 10. Insertion loss measurement setup for compressed and uncompressed urethane foam in double-wall structures.

Figure 11 shows the detailed setup for the measurement. The double wall has a lower base plate made of steel plate and an upper cover plate made of acrylic plate, sandwiching a porous layer between them.

The dimensions of the steel base plate and upper cover plate are the same as the outer dimensions of the heavy steel frame. The outer dimensions of the hard rubber spacer are the same as the outer dimensions of the frame. The inner dimensions of the spacer are the same as the inner dimensions of the frame. The size of the porous layer is also the same as the inner dimensions of the frame. The porous layer is made of the urethane foam, which is used around automobile body panels as sound insulation. The mass density of the urethane foam before compression is 65 kg/m³. The heavy steel

frame is placed on the upper cover plate. The weight of this frame compresses the porous layer until its thickness is equal to the thickness of the hard rubber spacer. As a result, the porous layer is compressed to 20mm in thickness from 30mm.

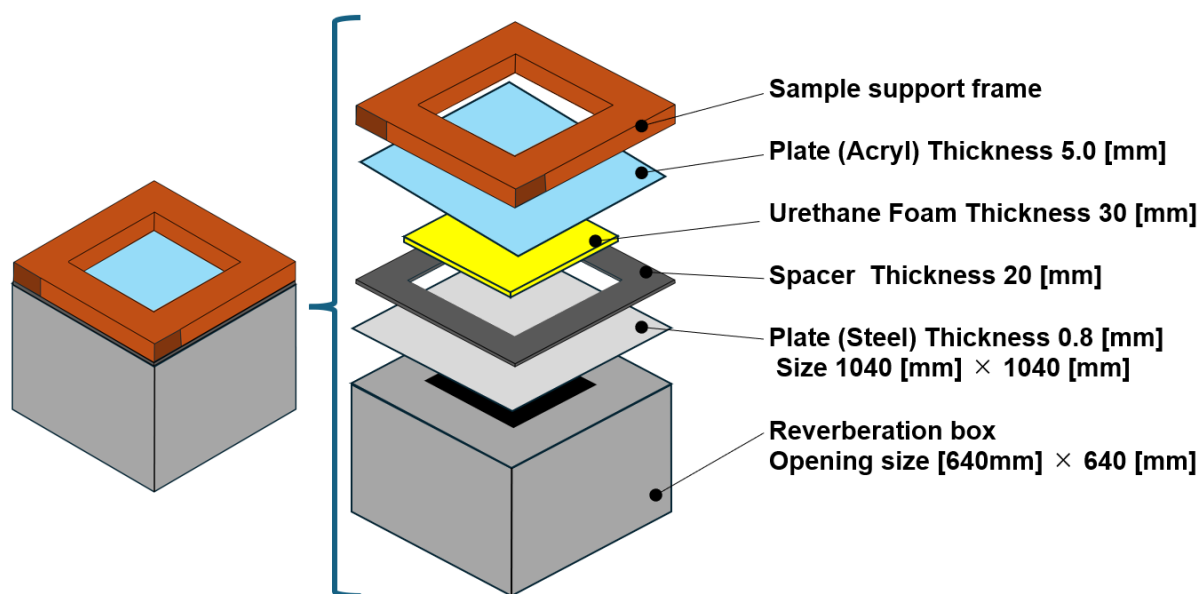


Fig. 11. Detailed setup for measurement of insertion loss.

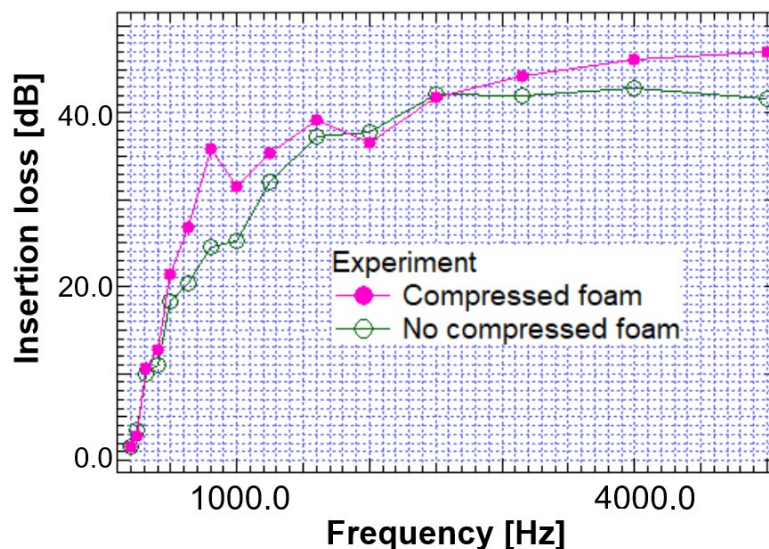


Fig. 12. Insertion loss for compressed and uncompressed urethane foam in double-wall structures (experiment).

Figure 12 presents the experimental results of insertion loss when urethane foam is placed between the layers of a double-wall structure. A comparison between the uncompressed and 33% compressed foam conditions indicates that compression improves the sound insulation performance. It is known that in the double walls with the foam material sandwiched in between the walls, the softer the foam material, the less wave transmission from the base plate to the cover plate, and therefore the better the sound insulation performance in the mid-to-high frequency range. Utilizing the softening effects, we succeeded to the increase insertion loss of 33% compressed urethane foam.

4. Conclusion

To investigate the softening mechanism of urethane foam under dynamic compressive loading, a nonlinear dynamic deformation analysis was conducted by applying the Kelvin model to the microcellular structure of the foam. As the dynamic compressive load increases and deformation progresses in the Z-direction, the beams within the microcell structure gradually tilt. This results in a reduction of axial deformation (associated with high stiffness) and an increase in bending deformation (associated with low stiffness). Consequently, the overall cell structure becomes more flexible, and the macroscopic foam exhibits a gradually softening restoring force characteristic. Therefore, it was concluded that the primary cause of the softening observed in urethane foam after 33% compression is geometric nonlinearity. This phenomenon is consistent with the quasi-static conditions as previous researchers were clarified. Using this softening effect, we succeeded to increase insertion loss of 33% compressed urethane foam sandwiched between double walls.

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