Impact Responses by FEM for a Living Finger Protected by Shock Absorbers Using Nonlinear Complex Springs Connected in Series

Toru Fukushima[1, a], Takao Yamaguchi[1,b], Yusaku Fujii[1,c], Shinichi Maruyama [1,d], Akihiro Takita[1,e] and Hisanori Tomita[1,f]

[1]Gunma University, 1-5-1 Tenjin-cho, Kiryu, Gunma, 376-8515 Japan

[a]<t14806006@gunma-u.ac.jp>, [b]<yamagme3@gunma-u.ac.jp>, [c]<fujii@gunma-u.ac.jp>, [d]<maruyama@gunma-u.ac.jp>, [e]<takita@gunma-u.ac.jp>, [f]<t08302054@gunma-u.ac.jp>

Keywords: finite element method, nonlinear vibration, viscoelastic material. nonlinear complex spring

Abstract. This paper deals with dynamic responses of a finger protected by viscoelastic absorbers under impact forces. Restoring forces of a finger and absorbers are measured using Levitation Mass Method proposed by Fujii. In this paper, we carry out numerical analysis of dynamic responses for the finger protected by the absorbers under same conditions with the experiment using LMM. The absorbers and the finger are modeled by nonlinear concentrated springs using power series of their elongations. We propose that the springs have nonlinear complex spring constants to represent changes of the hysteresis as their elongations progress. A finite element for the nonlinear complex spring is newly expressed using the relative displacement between the nodes on the ends of the spring. The nonlinear complex spring for the finger is inserted between the nonlinear complex springs for the two absorbers. These nonlinear complex springs are connected in series. Further, these nonlinear complex springs are attached to the levitated block, which is modeled by three-dimensional finite elements. By colliding the block with the finger inserted between absorbers, transient responses of this system are computed. The experimental data are compared with the calculated ones using our proposed FEM.

1. Introduction

We often use isolation using concentrated springs to protect structures (e.g. electronic apparatus, medical apparatus and precision apparatus) from undesirable impacts and external vibrations.

But, these lightweight structures sometimes do not have high rigidity. In such cases, these structures are regarded as elastic bodies. Some concentrated springs as isolating elements have nonlinear relations between load and displacement. Thus it’s important to clarify dynamics for the coupled problem between elastic bodies and nonlinear springs.

Many researchers have investigated for the nonlinear vibrations of concentrated masses with concentrated springs. For instance, Feeny studied this type of system using the proper orthogonal modes (POM) technique [1]. The dynamic responses for large-scale problems, which consisted of beams supported by nonlinear concentrated springs were studied by Kondou. Further, Kondou also proposed an efficient identification method for the nonlinear stability in large-scale problems [2]. Shaw proposed nonlinear modal analysis, and he applied it to a simply supported beam connected to a nonlinear concentrated spring [3]. Previously, we proposed a fast numerical method to calculate the nonlinear vibrations in an elastic block or a viscoelastic block with a nonlinear concentrated spring. In this study, one end of the nonlinear spring is attached to the ground [4,5]. Moreover, we extended the proposed method to deal with the dynamic phenomena of viscoelastic blocks connected with a nonlinear concentrated spring having a restoring force expressed as functions of the relative displacement between the two ends of the spring [6]. In the numerical analysis, the discrete equations in physical coordinate are transformed into the nonlinear ordinary coupled equations using normal coordinate corresponding to linear eigenmodes. In this calculation, modal damping is also transformed. The transformed equations are integrated numerically in extremely small degree-of freedom.

Some shock absorbers often have nonlinear hysteresis in their dynamic characteristics. For example, we observed phenomena that the hysteresis increases as the elongation of the shock absorbers increase. However, there are few reports to study coupled vibration between elastic structures and the shock absorbers having nonlinear hysteresis.

We extend our proposed method to deal with vibration analysis using finite element method for elastic structures including nonlinear concentrated spring with nonlinear hysteresis [7]. The restoring force of the spring is expressed as power series of its elongation. The restoring force also involves nonlinear hysteresis damping. Thus, complex stiffness is introduced for not only the linear component but also nonlinear components of the restoring force. Finite element for the spring is expressed and is attached to elastic structures modeled by linear solid finite elements. The discrete equations in physical coordinate are transformed into the nonlinear ordinary coupled equations using normal coordinate corresponding to linear eigenmodes. In this calculation, modal damping is also transformed. The transformed equations are integrated numerically in small degree-of freedom. This numerical method is applied for a finger protected by absorbers under impact forces as follows.

To protect human bodies such as arms [8,9], fingers and legs from impacts in mechanical apparatuses, buildings or transporters when doors are closed, viscoelastic rubbers are sometimes inserted between the doors and the frames around the doors. The rubbers have rolls of shock absorbers to decrease the impacts using viscoelasticity. These rubbers have many kinds of shapes to improve the performance of shock absorbers. In one of them, there are thin rubbers having hollow cross sections. For these types of the shock absorbers, buckling phenomena are sometimes utilized to decrease impacts efficiently. Thus, viscoelastic absorbers have nonlinear restoring force under relatively large load. The viscoelastic absorbers sometimes have nonlinear hysteresis in their dynamic behaviors. Thus, it's of importance to clarify nonlinear dynamic characteristics of viscoelastic shock absorbers with elastic structures under impact load. We apply our proposed method using Finite Element Method with the nonlinear complex springs to this problem. In the numerical method, the viscoelastic absorbers are modeled as the nonlinear springs with nonlinear hysteresis. The restoring force of the spring is expressed as power series of its elongation. The restoring force also involves nonlinear hysteresis damping. By colliding a block with a pair of viscoelastic shock absorbers with/without a living finger, velocities of the block are observed using Levitation Mass Method proposed by Fujii [7]. The experimental data [7] are compared with the calculated results using our proposed FEM.

2. Outline of Experimental Results

Using the Levitation Mass Method, velocity of the block is observed by colliding a block with a pair of viscoelastic shock absorbers. A living finger is set between the two absorbers. And the velocity of the block is also observed. Using the velocities, we studied the restoring forces.

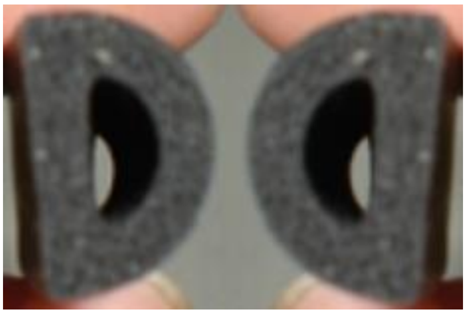

Fig. 1 A pair of viscoelastic shock absorbers

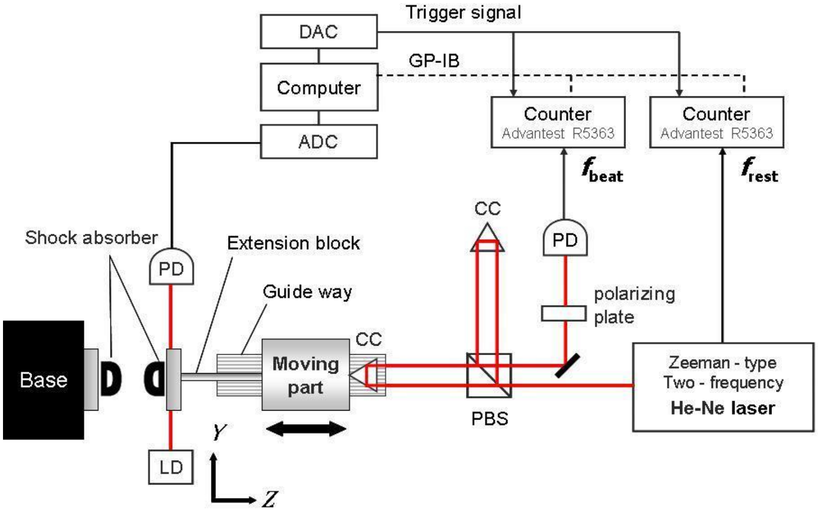

Fig.2 Experiment setup [7]

Figure 1 shows the cross sections of the pair of the viscoelastic absorbers. The shape of the absorber is tube with a thickness of 3[mm]. As shown in Fig.1, the cross sections of the absorber have D-type shapes. The absorbers are made of foam rubber. Figure 2 shows outline of the experimental setup using Levitation Mass Method proposed by Fujii [7]. This experimental system has a block, which can smoothly move along a guide in z direction using a pneumatic linear bearing. Therefore, the block is levitated by air film having 8[μm] thickness at the interfaces between the guide and the block. By using this system, the block can travel toward z direction under low friction. One of the viscoelastic shock absorbers is connected with a rigid base. Another viscoelastic absorber is attached to the levitated block. The block is pushed in negative z direction from the initial position z_0=0 by a human hand. Then, with initial velocity v_0, the block is collided with the pair of the viscoelastic shock absorbers. After the collision, velocity of the transient response for the block is observed using an optical interferometer. A corner cube is fabricated on the levitated block to receive laser beam for the interferometer. Acceleration of the block can be calculated by differentiation of the observed velocity. By integrating the measured velocity, position of the block is also determined. Inertia force can be calculated by product of the mass of the block and acceleration. And the inertia force is converted to restoring force by changing its sign. Then, hysteresis curve can be obtained using the relation between the position and the restoring force.

Figure 3 shows observed restoring forces. The ordinate is restoring force, and abscissa shows displacement. The red and green lines show the results of the absorbers without the finger. Under small initial velocity, the restoring force in for the red line has a characteristic of softening type spring. And its hysteresis in the restoring force is increasing as the displacement changes from 0[mm] to –5[mm]. Under condition of large initial velocity, the slope of the green curve suddenly changes to have harder spring constant when the displacement is –9[mm]. While the displacement progresses from –9[mm] to –10[mm], the hysteresis in the green line decrease. These behaviors are caused by the buckling phenomena of the shock absorbers. Note that there is no permanent deformation when the hollow parts of the D-shaped absorbers are buckled. The blue line represents the result of the shock absorbers with the human finger. The slope of this curve also changes when the displacement is about –9[mm]. The blue curve also shows soft-hardening characteristics in the restoring force. While the hysteresis decreases from –9[mm] to –14[mm], the hysteresis in the blue line increases from 0[mm] to -9[mm]. The dynamic characteristics of the absorbers with/without the finger are similar. But, the restoring force for the viscoelastic absorbers without the finger has sharper change than that for the absorbers with the finger. We regard that this phenomenon is due to elastic deformations of the living finger.

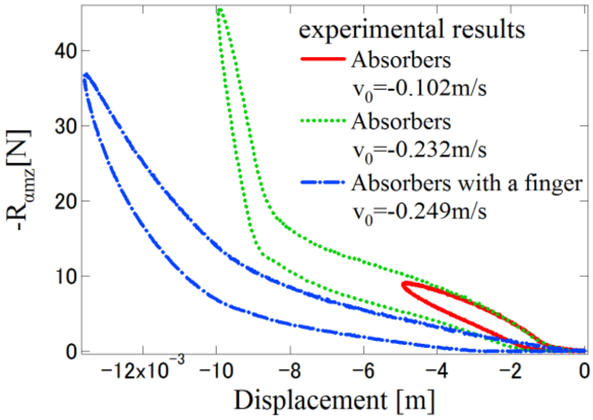

Fig.3 Experimental results of restoring forces for viscoelastic shock absorbers

3. Numerical analysis

Considering the experimental results in the previous chapter, the following behaviors for the restoring forces of the viscoelastic absorbers with/without the living finger are summarized.

(a) There are behaviors as nonlinear springs.

(b) The hysteresis increases when the deformation of the springs become large. The hysteresis inversely decreases after the deformation become larger.

(c) Near the regular deformations, the slopes of the curves in the restoring forces increase.

The phenomena (a) and (b) are common behaviors for the absorbers with/without the finger. The phenomenon (c) appears the viscoelastic absorbers with/without the finger under large deformation. The change of the stiffness for the absorbers with the finger is smaller than that for the viscoelastic absorbers without the living finger.

To simulate these phenomena appeared in the experiment, we use the FEM model as shown in Fig.4. We use two nonlinear concentrated springs having nonlinear hysteresis for the pair of the viscoelastic shock absorbers and one nonlinear spring for the finger to simulate the restoring forces in Fig.3. We propose a finite element for springs having nonlinear complex spring constants. The nonlinear complex spring of the finger is connected in series between the two nonlinear complex springs of the absorbers. These springs are attached to the levitated block using three-dimensional solid finite elements. Detail numerical procedures are shown in the following sections.

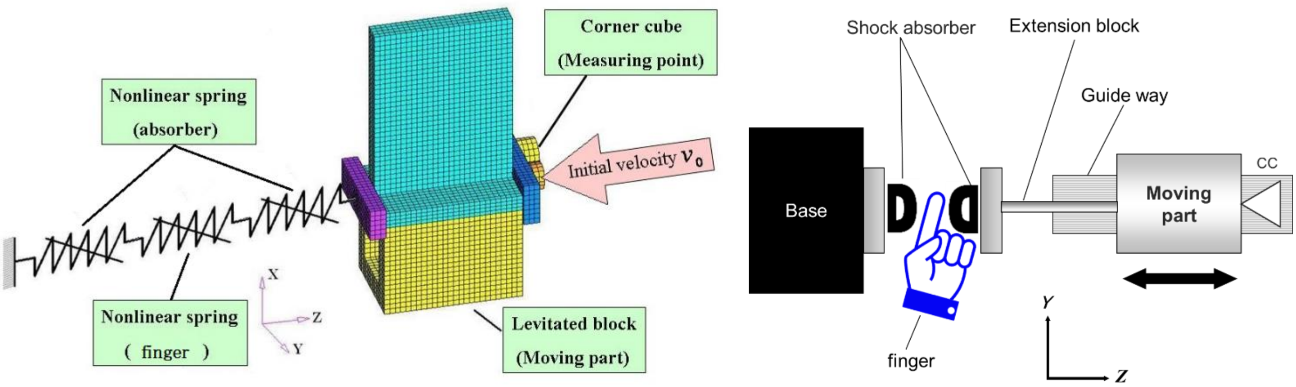

Fig.4 Simulation model

3.1 Discrete Equation for Nonlinear Complex Concentrated Springs (Viscoelastic Shock Absorbers and a Living Finger)

The pair of the viscoelastic shock absorbers is modeled by using two concentrated nonlinear springs with nonlinear hysteresis. For the finger, we also use another nonlinear spring with nonlinear hysteresis. We assume that the *m*-th nonlinear concentrated spring with viscoelasticity for them has principal elastic axis in z direction. The *m*-th nonlinear concentrated spring is connected between the nodal points α_m and β_m. We denote displacements as $U_{\alpha m z}$ and $U_{\beta m z}$ in z direction at the nodal points α_m and β_m, respectively, Nonlinear restoring force $R_{\alpha m z}$ of the *m*-th spring using power series is given for nodal force at the point α_m. Thus, the restoring force $R_{\alpha m z}$ is expressed using the relative displacement $(U_{\alpha m z} - U_{\beta m z})$ as $R_{\alpha m z} = \gamma_{1mz}(U_{\alpha m z} - U_{\beta m z}) + \gamma_{2mz}(U_{\alpha m z} - U_{\beta m z})^2 + \gamma_{3mz}(U_{\alpha m z} - U_{\beta m z})^3 + \cdots$, $R_{\beta m z} = -R_{\alpha m z}$,, $R_{\alpha m x} = R_{\alpha m y} = R_{\beta m x} = R_{\beta m y} = 0$. Further, linear hysteresis damping is introduced as $\gamma_{1mz} = \bar{\gamma}_{1mz}(1 + j\eta_{1mz})$. $\bar{\gamma}_{1mz}$ is the real part of γ_{1mz}, and η_{1mz} is the material loss factor of the concentrated spring. j is the imaginary unit. Moreover, nonlinear hysteresis damping is also introduced as $\gamma_{2mz} = \bar{\gamma}_{2mz}(1 + j\eta_{2mz})$, $\gamma_{3mz} = \bar{\gamma}_{3mz}(1 + j\eta_{3mz})$ and $\gamma_{4mz} = \bar{\gamma}_{3mz}(1 + j\eta_{4mz})$ in the same manner. $\bar{\gamma}_{2mz}, \bar{\gamma}_{3mz}$ and $\bar{\gamma}_{4mz}$ are the real part of $\gamma_{2mz}, \gamma_{3mz}$ and γ_{4mz}, respectively. η_{2mz}, η_{3mz} and η_{4mz} are the nonlinear components of material loss factor for the concentrated spring, respectively. These relations can be rewritten in the matrix form as:

$$\{R_m\} = [\bar{\gamma}_{1m}]\{U_{sm}\} + \{\bar{d}_m\} \tag{1}$$

Where, $\{R_m\}$ is the nodal force vector at the node α_m. $\{U_{sm}\}$ is the nodal displacement vector at the node α_m. $[\bar{\gamma}_{1m}]$ is the complex stiffness matrix including only linear term of the restoring force. $\{\bar{d}_m\}$ is the vector involving nonlinear terms of the restoring force.

3.2 Discretized equations of an elastic structure

It is assumed that equations of motion for the levitated block are expressed under small deformation. We consider that damping of the levitated block using complex modulus of elasticity $E = \bar{E}(1 + j\eta_b)$. The real part $\bar{E}$ of the E represents the Young's modulus of elasticity. η_b is the material loss factor of the block. All elements related to the structure are superposed, the equations in the entire domain of the levitated block are obtained as:

$$[M_b]\{\ddot{U}_b\} + [K_b]\{U_b\} = \{F_b\} \tag{2}$$

Where, $\{F_b\}$, $\{U_b\}$, $[M_b]$ and $[K_b]$ are the force vector, the displacement vector, the mass matrix and complex stiffness matrix, respectively. We mainly use isoparametric hexahedral elements with non- conforming modes [10,11] for the later numerical computation.

3.3 Discrete equation for combined system between a block and viscoelastic springs

The *m*-th restoring force $\{R_m\}$ in Eq. (1) for the nonlinear complex springs are added to the corresponding nodal forces at the nodal points α_m and β_m. We also add the restoring force at the attached node α between the nonlinear concentrated spring for one of the viscoelastic shock absorbers and the levitated block. The following equation in the global system can be obtained:

$$[M]\{\ddot{U}\} + [K]\{U\} + \{\hat{d}\} = \{F\} \tag{3}$$

Where, $\{U\}$, $\{F\}$, $[M]$ and $[K]$ are the displacement vector, the external force vector, the mass matrix and the complex stiffness matrix in the global system, respectively. $\{\hat{d}\}$ is the nonlinear complex restoring force of the nonlinear springs in series for the absorbers and the finger. $\{\hat{d}\}$ has the identical vector size to degree- of- freedom of Eq.(3).

3.4 Conversion from Discrete Equations in Physical Coordinate to Nonlinear Equations in Normal Coordinate

Long computational time requires to compute Eq. (3) in physical coordinate, directly. In this section, a numerical procedure is explained to decrease the degree-of-freedom for the discrete equations of motion.

First, it is assumed that linear eigenmodes $\{\phi^{(i)}\}$ of vibration can be approximated to the real eigenmodes $\{\phi^{(i)}\}_0$. Next, by introducing normal coordinates $\tilde{b}_i$ corresponding to the linear eigenmodes $\{\phi^{(i)}\}_0$, the nodal displacement vector can be expressed using both $\{\phi^{(i)}\}_0$ and $\tilde{b}_i$ [12].

$$\{U\} = \sum_{i=1} \tilde{b}_i \{\tilde{\phi}^{(i)}\}_0 \tag{4}$$

Substituting of Eq.(4) into Eq.(3), the following nonlinear ordinary simultaneous equations as to normal coordinates $\tilde{b}_i$ can be expressed as:

$$\ddot{\tilde{b}}_i + \eta_{tot}^{(i)}\omega_0^{(i)}\dot{\tilde{b}}_i + \left(\omega_0^{(i)}\right)^2 \tilde{b}_i + \sum_j\sum_k \tilde{D}_{ijk}\tilde{b}_j\tilde{b}_k + \sum_j\sum_k\sum_l \tilde{E}_{ijkl}\tilde{b}_j\tilde{b}_k\tilde{b}_l + \cdots - \tilde{P}_i = 0, \quad i,j,k,l = 1,2,3,\ldots \tag{5}$$

$$\tilde{P}_i = \{\tilde{\phi}^{(i)}\}_0^T\{F\}, \quad \{\tilde{\phi}^{(i)}\}_0 = \{\tilde{\psi}_{1ix}, \tilde{\psi}_{1iy}, \tilde{\psi}_{1iz}, \tilde{\psi}_{2ix}, \tilde{\psi}_{2iy}, \cdots, \tilde{\psi}_{\alpha mix}, \tilde{\psi}_{\alpha miy}, \tilde{\psi}_{\alpha miz}, \cdots, \tilde{\psi}_{\beta mix}, \tilde{\psi}_{\beta miy}, \tilde{\psi}_{\beta miz}, \cdots\}^T,$$

$$\tilde{D}_{ijk} \equiv \sum_{m=1} \gamma_{2mz}\left(\tilde{\psi}_{\alpha miz} - \tilde{\psi}_{\beta miz}\right)\left(\tilde{\psi}_{\alpha mjz}\tilde{\psi}_{\alpha mkz} - \tilde{\psi}_{\alpha mjz}\tilde{\psi}_{\beta mkz} - \tilde{\psi}_{\beta mjz}\tilde{\psi}_{\alpha mkz} + \tilde{\psi}_{\beta mjz}\tilde{\psi}_{\beta mkz}\right),$$

$$\begin{aligned}\tilde{E}_{ijkl} \equiv \sum_{m=1} \gamma_{3mz}\left(\tilde{\psi}_{\alpha miz} - \tilde{\psi}_{\beta miz}\right)&\left(\tilde{\psi}_{\alpha mjz}\tilde{\psi}_{\alpha mkz}\tilde{\psi}_{\alpha mlz} - \tilde{\psi}_{\alpha mjz}\tilde{\psi}_{\alpha mkz}\tilde{\psi}_{\beta mlz} - \tilde{\psi}_{\alpha mjz}\tilde{\psi}_{\beta mkz}\tilde{\psi}_{\alpha mlz} - \tilde{\psi}_{\beta mjz}\tilde{\psi}_{\alpha mkz}\tilde{\psi}_{\alpha mlz}\right.\\ &\left. + \tilde{\psi}_{\alpha mjz}\tilde{\psi}_{\beta mkz}\tilde{\psi}_{\beta mlz} + \tilde{\psi}_{\beta mjz}\tilde{\psi}_{\alpha mkz}\tilde{\psi}_{\beta mlz} + \tilde{\psi}_{\beta mjz}\tilde{\psi}_{\beta mkz}\tilde{\psi}_{\alpha mlz} - \tilde{\psi}_{\beta mjz}\tilde{\psi}_{\beta mkz}\tilde{\psi}_{\beta mlz}\right)\end{aligned}$$

In Eq. (5), $\omega_0^{(i)}$ is the *i*-th natural frequency. $\eta_{tot}^{(i)}$ is the *i*-th modal loss factor. Because Eq. (5) has much smaller degree-of -freedom than that of Eq. (3), we can save computational time. In Eq. (5), dot stands for partial differentiation with respect to time *t*. $\tilde{\psi}_{\alpha miz}$ is the *z*-component of the eigenmode $\{\tilde{\phi}^{(i)}\}_0$ at the attached nodes α_m of the nonlinear complex springs.

3.5 Nonlinear Impact Response

Nonlinear impact responses are computed using Runge-Kutta-Gill method to Eq.(5). In the numerical integration, we give an impact for the force vector $\{F\}$ in Eq. (5).

4. Numerical results and discussion

The same initial velocities as the experimental values in Fig.3 are given to the simulation model of the levitated block. We computed the restoring forces using the simulation model as shown in Fig.4 by colliding the block with the viscoelastic shock absorbers with/without the finger. In this figure, the levitated block is modeled as an elastic body using finite elements. The pair of the viscoelastic shock absorbers and the finger are modeled using nonlinear complex springs to consider nonlinear damping. We set an origin of this model on the position where the levitated block begins to contact with the pair of the absorbers with/without the finger. Firstly, the real part of the nonlinear spring constants are identified using backbone curves from the experiment. Next, we determine the linear/nonlinear loss factors in the proposed nonlinear complex spring constants by fitting curves of the nonlinear restoring forces.

4.1 Numerical Results for a Pair of Shock Absorbers without a Finger

The following nonlinear complex spring constants are identified for the restoring forces of each absorber of two without the finger. We use the nonlinear spring constants from the first to the seventh components of power series.

$\bar{\gamma}_{1mz} = 1.00\times10^{3}$ [N/m], $\eta_{1mz} = 0.20$ [-], $\bar{\gamma}_{2mz} = 1.05\times10^{6}$ [N/m^2], $\eta_{2mz} = 0.05$ [-],
$\bar{\gamma}_{3mz} = 2.20\times10^{9}$ [N/m^3], $\eta_{3mz} = 0.06$ [-], $\bar{\gamma}_{4mz} = 6.80\times10^{11}$ [N/m^4], $\eta_{4mz} = 0.06$ [-],
$\bar{\gamma}_{5mz} = 3.30\times10^{13}$ [N/m^5], $\eta_{5mz} = 0.05$ [-], $\bar{\gamma}_{6mz} = 0.00$ [N/m^6], $\eta_{6mz} = 0.00$ [-],
$\bar{\gamma}_{7mz} = 1.50\times10^{18}$ [N/m^7], $\eta_{7mz} = 0.09$ [-]

Figure 5 shows the calculated restoring forces of the pair of the viscoelastic shock absorbers without the finger in comparison with the experimental ones. In the figure, the red curves show the results under small initial velocity, while green curves show the results under the large initial velocity. Both results are consistent qualitatively between the experimental curves and the calculated curves. In special, the slopes of the experimental/calculated curves increase near the deformation – 9[mm]. The changes of the hysteresis in the curves can also be computed. Note that these characteristics cannot be reproduced using the only linear term of hysteresis damping. We can confirm the consistency between the experiment and the numerical analysis by introducing the nonlinear complex springs in series.

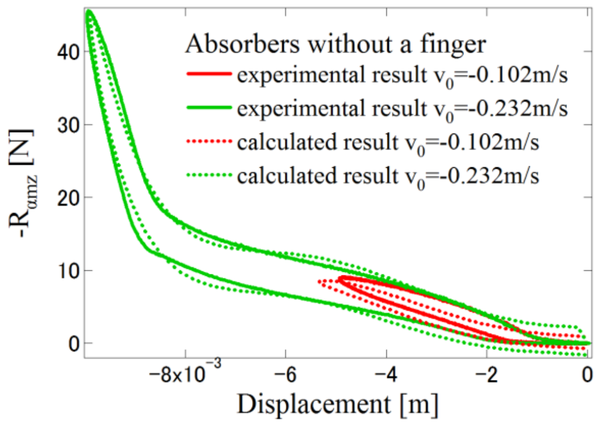

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Fig. 5 Hysteresis curves of a pair of viscoelastic shock absorbers without a finger from FEM

4.2 Calculated Results for a Pair of Shock Absorbers with a Finger

Next, to determine the parameters for the living finger, we compute the restoring forces of the shock absorbers with the finger by varying the nonlinear complex spring constants of the finger. For this computation, we used the previously determined nonlinear complex spring constants for the absorbers. We connected these nonlinear complex springs in series as shown in Fig.4. Finally, we identified the following nonlinear complex spring constants for the restoring forces of the living finger. We use the nonlinear spring constants from the first to the seventh components of power series as follows.

$\bar{\gamma}_{1mz} = 7.50\times10^{2}$ [N/m], $\eta_{1mz} = 0.20$ [-], $\bar{\gamma}_{2mz} = 0.00$ [N/m^2], $\eta_{2mz} = 0.00$ [-],
$\bar{\gamma}_{3mz} = 0.00$ [N/m^3], $\eta_{3mz} = 0.00$ [-], $\bar{\gamma}_{4mz} = 0.00$ [N/m^4], $\eta_{4mz} = 0.00$ [-],

$\bar{\gamma}_{5mz} = 4.50 \times 10^{12}$ [N/m^5], $\eta_{5mz} = 0.10$ [-], $\bar{\gamma}_{6mz} = 0.00$ [N/m^6], $\eta_{6mz} = 0.00$ [-],
$\bar{\gamma}_{7mz} = 3.40 \times 10^{17}$ [N/m^7], $\eta_{7mz} = 0.10$ [-]

Figure 6 shows comparison between the experimental restoring curve of the shock absorbers with the living finger under an initial velocity =-0.249[m/s] and calculated ones using these parameters. As can be seen in the Fig 6, the computed and the experimental curves agree well. Consequently, we confirm that our proposed method is valid.

The changes of the slopes appear both in the experimental and calculated curves near the deformation = -9.0[mm]. These changes of the slopes are less than those in the restoring forces of the absorbers without the finger discussed in the previous section 4.1. We think this phenomenon is due to the elasticity and viscoelasticity of the living finger.

The rigidity of the finger is larger than the viscoelastic shock absorbers under small deformation. Initially, the pair of the soft viscoelastic absorbers deforms, and then the absorbers are compressed. Due to the compression, the stiffness of the absorbers become larger than the stiffness of the finger. Then, the finger deforms.

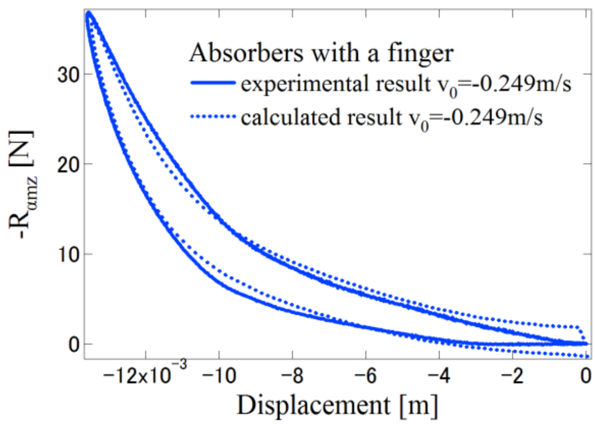

Fig. 6 Hysteresis curves of a pair of viscoelastic shock absorbers with a finger from FEM

4.3 Numerical analysis to study dynamic errors of experimental system using LMM

In this section, to check dynamic errors of the levitated block, we study accuracy of the measurement system using the Levitation Mass Method. To study undesirable dynamic behaviors for this system, we investigated consistency between displacement D_c at the corner cube and the displacement D_g at the center of gravity of the block. From the viewpoint of high precision measurement, the displacements D_c and D_g must have the same values. If rigid motions except the motions in *z* direction or elastic deformations of the block increase, the undesirable differences between D_c and D_g will increase. We obtained the ratio $D_c / D_g = 1.0000002175737$ when the displacements reach up to the local maximal values at *t*=0.600 [sec]. From this ratio, we can regard that the undesirable dynamic components of the levitated block are negligible small. Thus, we confirm that all of the restoring forces in chapter 4 are free from the dynamic errors due to the undesirable behaviors of the block in the measurement system.

5. Conclusion

By introducing the proposed complex nonlinear spring elements connected in series into numerical analysis using FEM, we compute impact responses of a human living finger protected by viscoelastic shock absorbers. We can reproduced the experimental restoring forces of the pair of the shock absorbers with/without the finger. We can simulate the change of hysteresis due to their deformation.

Acknowledgements

This work was supported by Grant-in-Aid for Scientific Research (C) 26420167 (KAKENHI 26420167).

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