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A Study on Loop Gain Measurement Method Using Output Impedances in Operational Amplifier

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Abstract. This paper describes a method to derive the loop gain and phase from the output impedances in analog circuit with negative feedback. Especially, we extend our previously proposed method for a DC-DC buck converter, to the operational amplifier circuit with negative feedback. This enables to measure the loop gain without injecting a signal into feedback loop, i.e. without breaking the feedback loop; hence the proposed method can be applied for the operational amplifier circuit with negative feedback implemented on an IC. Our simulation results show that the loop gain obtained from the proposed method matches very well with the one from the conventional method.

1. Introduction

It is very important to evaluate the stability of the negative feedback circuit by measuring the loop gain in the target circuit. In general, the loop gain is often measured by the voltage injection method [1-3], which determines the loop gain by measuring the amplitude and phase differences between the AC voltage source injected into the feedback loop. However, the voltage injection method has the following problems:

1. This method needs to inject a voltage signal in series into the feedback loop with breaking a measurement point in the feedback loop.
2. An injection point has some circuit characteristics restrictions.

To overcome these problems, we have reported on the measurement method of the loop gain using the output impedance for a DC-DC buck converter [4, 5]. In this paper, we show that the proposed method is applicable also to operational amplifier circuits with negative feedback configuration. Measurement of the loop gain is important for the operational amplifier circuit system with negative feedback to evaluate its phase and gain margins.

2. Loop Gain Measurement Principle by Conventional Method

Figure 1 shows a transfer function block diagram of a negative feedback circuit using the Thévenin equivalent circuit model. The output port of Block 1 is represented by a Thévenin equivalent network, Z_1 is the output impedance of Block 1, which is loaded by the Z_2, the input impedance of Block 2. Let us consider the measurement of the loop gain at point A in Fig. 1. The transfer function from ΔV_x to ΔV_y is obtained as follows[3]:

$$T = G_1\left(\frac{Z_2}{Z_1 + Z_2}\right)G_2H \tag{1}$$

This is the true value of the loop gain T in Fig. 1.

Next, let us consider the measurement of the loop gain in Fig. 1 by the voltage injection method. Figure 2 shows a block diagram of the measurement of the loop gain T by the voltage injection method. To measure the loop gain by the voltage injection, we connect a network analyzer to measure the transfer function T_v from ΔV_x to ΔV_y. The AC input source is injected between Block 1 and Block 2 in series, and the injection voltage ΔV_Z is swept over the intended frequency range. The measured gain T_v is given by [3]:

$$T_v \equiv \left.\frac{\Delta V_y}{\Delta V_x}\right]_{\Delta V_{ref}=0, \Delta V_{in}=0} \tag{2}$$

Solution of Fig. 2 for the measured gain T_v is obtained as follows:

$$T_v = T\left(1 + \frac{Z_1}{Z_2}\right) + \frac{Z_1}{Z_2} \tag{3}$$

Let Z_S be the output impedance of the injection voltage, and it can be ignored by assuming $Z_S \ll Z_2$. T_v is approximately equal to T in case $Z_1 \ll Z_2$. The above is the measurement principle of the loop gain by the voltage injection method.

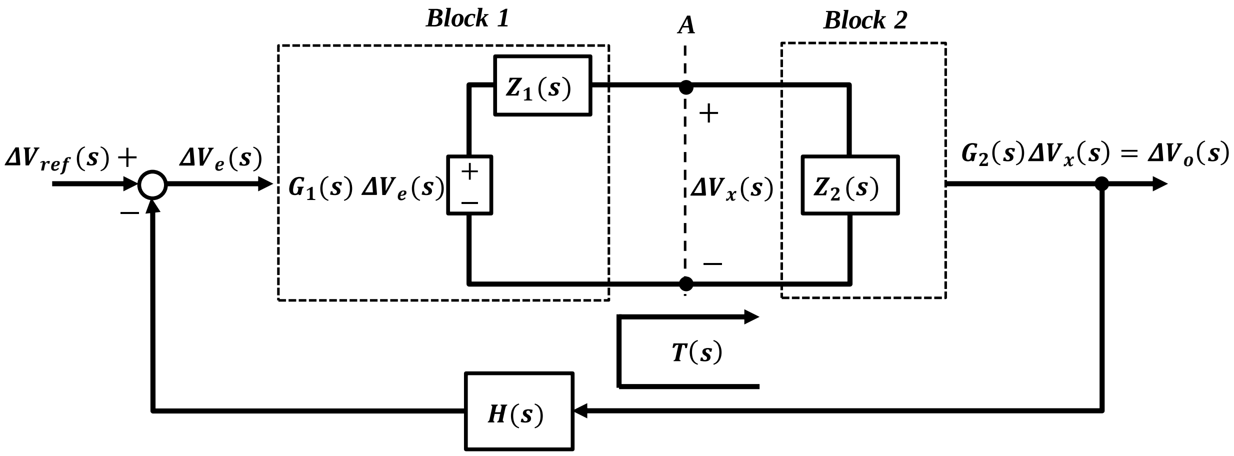


Fig. 1. Functional block diagram of a typical negative feedback circuit represented by the Thévenin equivalent circuit [3].

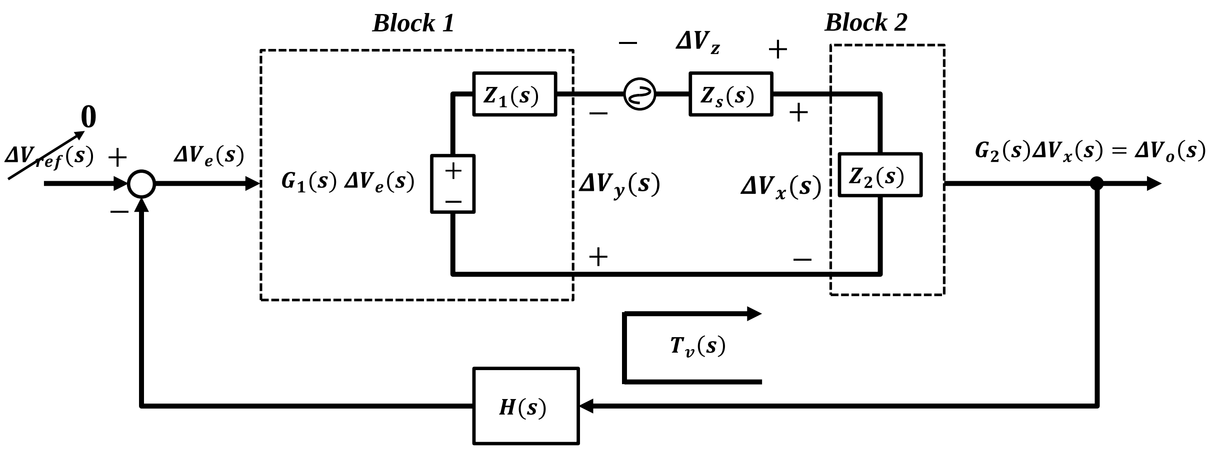


Fig. 2. Measurement of the loop gain by the voltage injection [3].

3. Measurement Principle of Loop Gain by Proposed Method

3.1 Derivation using Thévenin Equivalent Circuit

The loop gain is calculated from the output impedances in open and close loop conditions. First, we explain the measurement of Z_O, the output impedance in open loop condition. Figure 3 shows a block diagram of measurement of the output impedance in open loop. Z_O is defined as

$$Z_o \equiv \left.\frac{\Delta V_x}{\Delta I_z}\right]_{\Delta V_{in}=0, \Delta V_e=0} \tag{4}$$

When $\Delta V_e = 0$, the change is not transmitted to the feedback side, even if the output voltage ΔV_O fluctuates by the injection of AC current ΔI_Z. It means that the circuit in Fig.3 operates in open loop condition. Solution of Fig. 3 for the Z_O is obtained as follows:

$$Z_o = \frac{Z_1 Z_2}{Z_1 + Z_2} \tag{5}$$

Next, let us explain about the measurement of Z_{oc}, the output impedance in close loop condition. Figure 4 shows a block diagram of measurement of the output impedance in closed loop. Z_{oc} is defined as

$$Z_{oc} \equiv \left.\frac{\Delta V_x}{\Delta I_z}\right]_{\Delta V_{in}=0, \Delta V_{ref}=0} \tag{6}$$

When Kirchhoff's current law is applied to the injection point of AC current ΔI_Z in Fig. 4, we have

$$\Delta I_z = \frac{\Delta V_x}{Z_2} + \frac{\Delta V_x - (-\Delta V_x H G_1 G_2)}{Z_1} \tag{7}$$

Equations (1) and (5) are substituted for Eq. (7). Also Equation (7) is solved for Z_{oc}.

$$Z_{oc} = \frac{\frac{Z_1 Z_2}{Z_1 + Z_2}}{1 + G_1\left(\frac{Z_2}{Z_1 + Z_2}\right) G_2 H} = \frac{Z_o}{1 + T} \tag{8}$$

Equation (8) is solved for the loop gain T.

$$T(s) = \frac{Z_o - Z_{oc}}{Z_{oc}} \tag{9}$$

The Bode plots are drawn by Eqs. (10) and (11), because $T(s)$ in Eq. (9) is a complex function.

$$20\log_{10}|T| = 20\log_{10}\left(\frac{|Z_o - Z_{oc}|}{|Z_{oc}|}\right) \tag{10}$$

$$arg(T) = arg(Z_o - Z_{oc}) - arg(Z_{oc}) \tag{11}$$

We can see from Eqs.(10) and (11) that the gain and phase of loop gain in negative feedback circuit can be measured from Z_o and Z_{oc}.

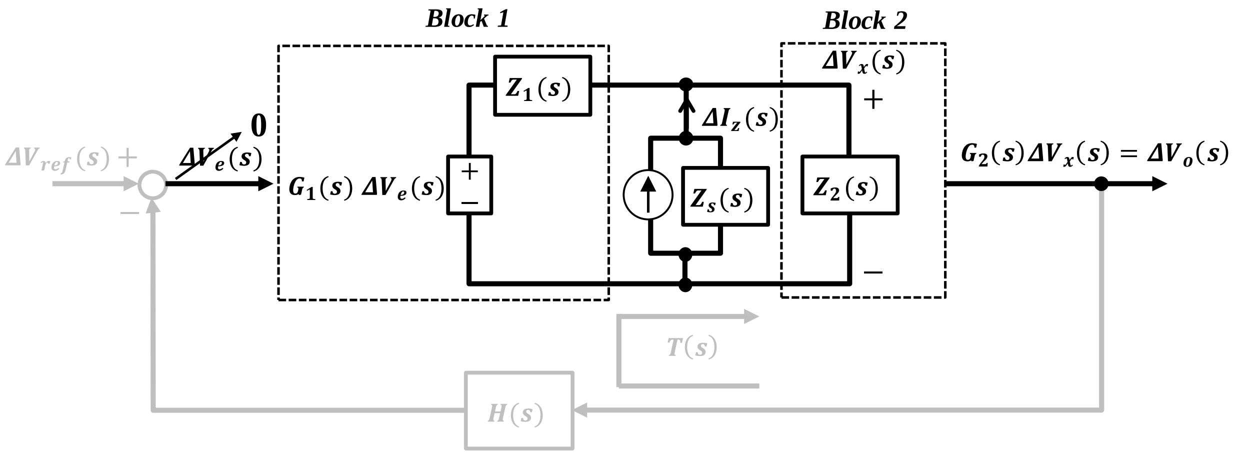


Fig. 3. Measurement of the output impedance in open loop.

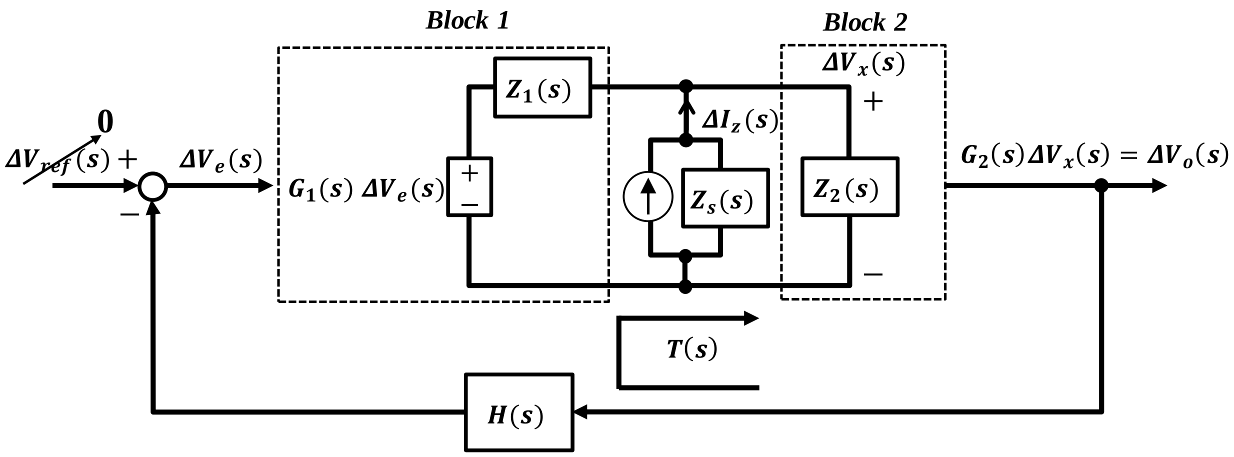


Fig. 4. Measurement of the output impedance in closed loop.

3. 2 Derivation Using Equivalent Circuit Model of Operational Amplifiers

Figure 5(a) shows a block diagram of measurement of the output impedance in open loop at non-inverting amplifier circuit. As defined in Eq. (4), it is required to measure the open loop output impedance under the condition of $\Delta V_e = 0$. In Fig. 5(a), the condition of $\Delta V_e = 0$ is realized by injection of a second order LC low pass filter in the feedback circuit. When Kirchhoff's current law is applied to the injection point of AC current ΔI_O in Fig. 5(a),

$$\Delta I_o = \frac{\Delta V_o}{R_o} + \frac{\Delta V_o}{R_1 + R_2} \tag{12}$$

Equation (12) is solved for $\Delta V_o/\Delta I_o$.

$$Z_o = R_o//(R_1 + R_2) = \frac{R_o(R_1 + R_2)}{R_o + R_1 + R_2} \tag{13}$$

Equation (13) means the open loop output impedance Z_o.

Figure 5(b) shows a block diagram of measurement of the output impedance in closed loop at non-inverting amplifier circuit. When Kirchhoff's current law is applied to the injection point of AC current ΔI_O in Fig. 5(b),

$$\Delta I_o + \frac{A\Delta v_e - \Delta V_o}{R_o} + \frac{\Delta V_o - \Delta v_e}{R_2} = 0 \tag{14}$$

From Fig. 5(a), the error voltage Δv_e is obtained as follows:

$$\Delta v_e = -\Delta V_o \frac{R_1//R_i}{R_1//R_i + R_2} \tag{15}$$

Equation (15) is substituted for Eq. (14), and it is solved for $\Delta V_o/\Delta I_o$.

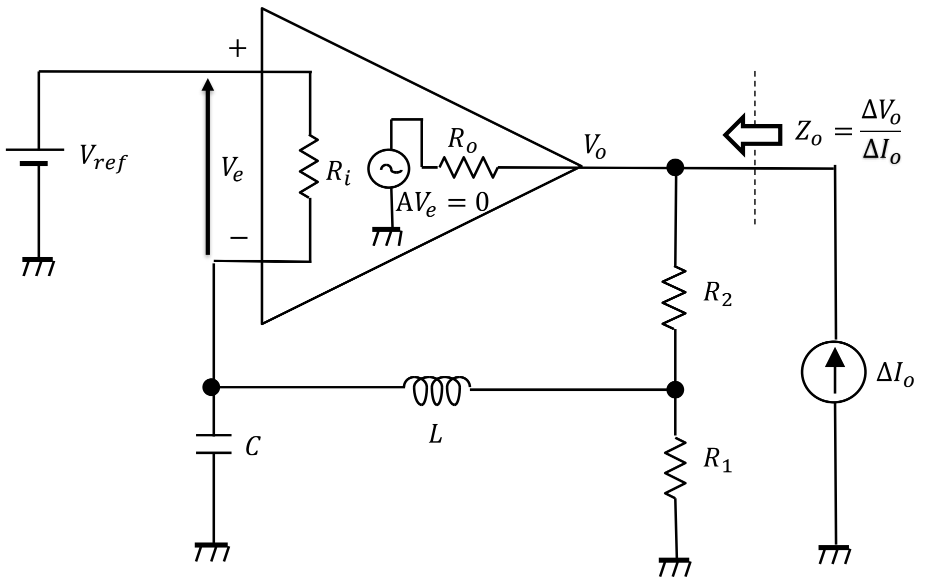
$$Z_{oc} = \frac{R_o\left(1 + \frac{R_2}{R_1} + \frac{R_2}{R_i}\right)}{1 + \frac{R_2}{R_1} + \frac{R_2}{R_i} + A + \frac{R_o}{R_1} + \frac{R_o}{R_i}} \tag{16}$$

Z_{oc} can be considered as follows, by assuming R_2, $R_o \ll R_i$:

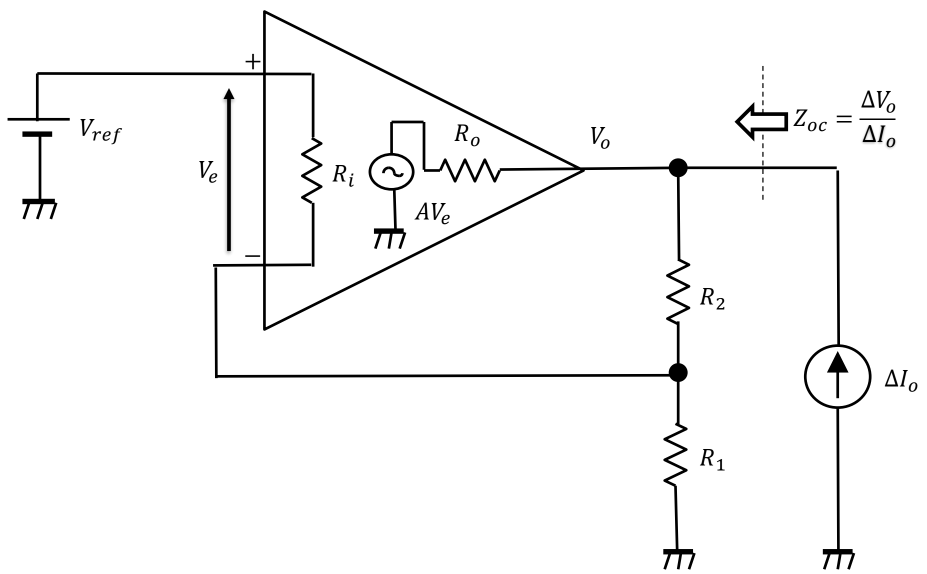
$$Z_{oc} \approx \frac{Z_o}{1 + A\beta} = \frac{Z_o}{1 + T} \tag{17}$$

where, $\beta = R_2/(R_1 + R_2)$ is feedback rate. Z_{oc} in Eq. (17) is the closed loop output impedance. It corresponds to Z_{oc} obtained by Eq. (8). It means that the principle of the loop gain measurement by the proposed method also is established at the circuit level.

In the proposed method, it is not necessary to inject an AC voltage source into the feedback circuit. Also the input / output impedance at the injection point of the AC current source does not affect the accuracy of the loop gain measurement.



(a) Measurement of Z_o



(b) Measurement of Z_{oc}

Fig. 5. Measurement of the output impedance in non-inverting amplifier circuit.

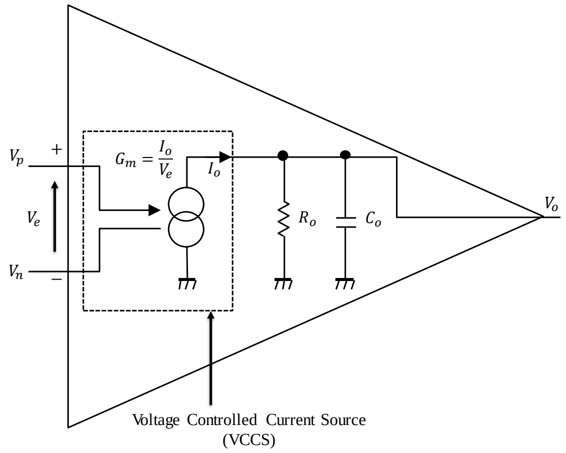
4. Simulation Verification

4.1 Simulation Conditions

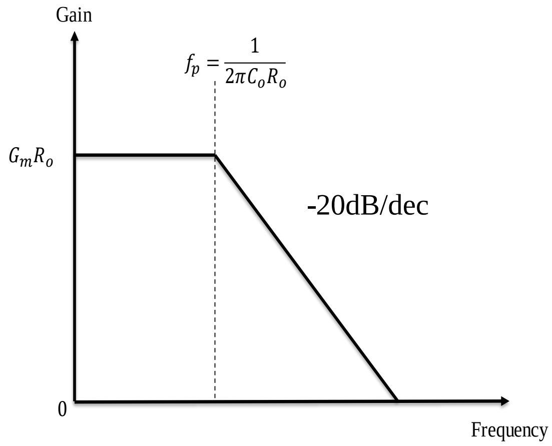
We have compared the simulated results of the loop gain measurements by the voltage injection method (conventional method) and the proposed method. The measurement circuit used in this simulation verification is a non-inverting amplifier circuit with the first-order lag model shown in Fig. 6. Using a voltage controlled current source (VCCS), this model can easily design the open loop gain and the primary pole of the operational amplifier. Here the transconductance of VCCS is G_m, the open loop gain is $\mathrm{A} = G_m R_o$, the primary pole is $f_p = 1/(2\pi C_o R_o)$, and the input impedance is infinity. The simulation circuits of the conventional method and the proposed method are shown in Figs. 7 and 8 respectively. We have conducted simulation using SIMPLIS simulator by SIMetrix Technologies Ltd. The simulation parameters for Figs. 7 and 8 are shown in Table 1.

Table 1. Simulation parameters

Parameter	Value	Parameter	Value
V_{ref}	1 V	R_1	9 kΩ
R_o	50 Ω	R_2	1 kΩ
C_o	0.64 μF	L_1	1 H
G_m	20 S	C_1	1 F



(a) Equivalent circuit model



(b) Frequency characteristic of its gain

Fig. 6. First-order lag model of an operational amplifier.

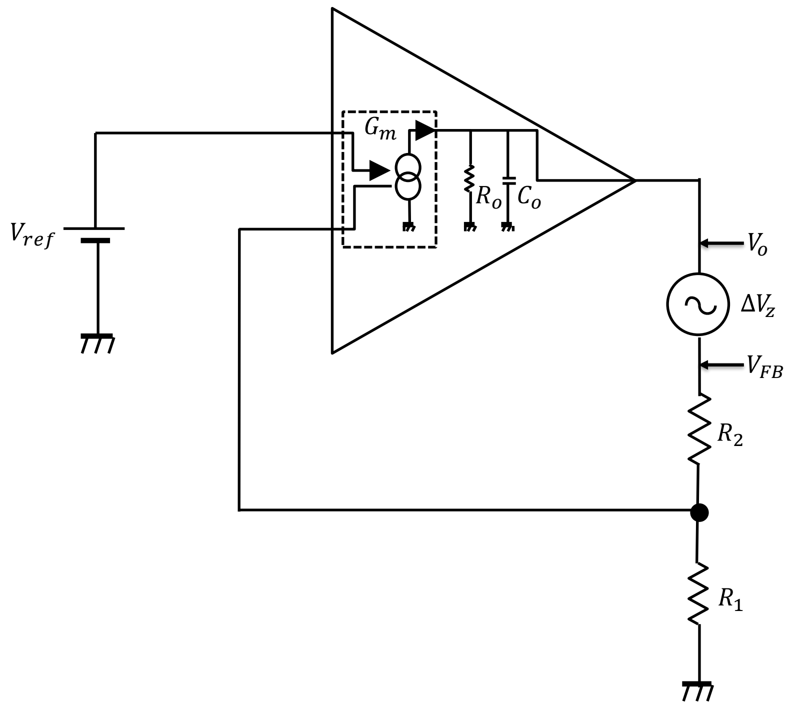
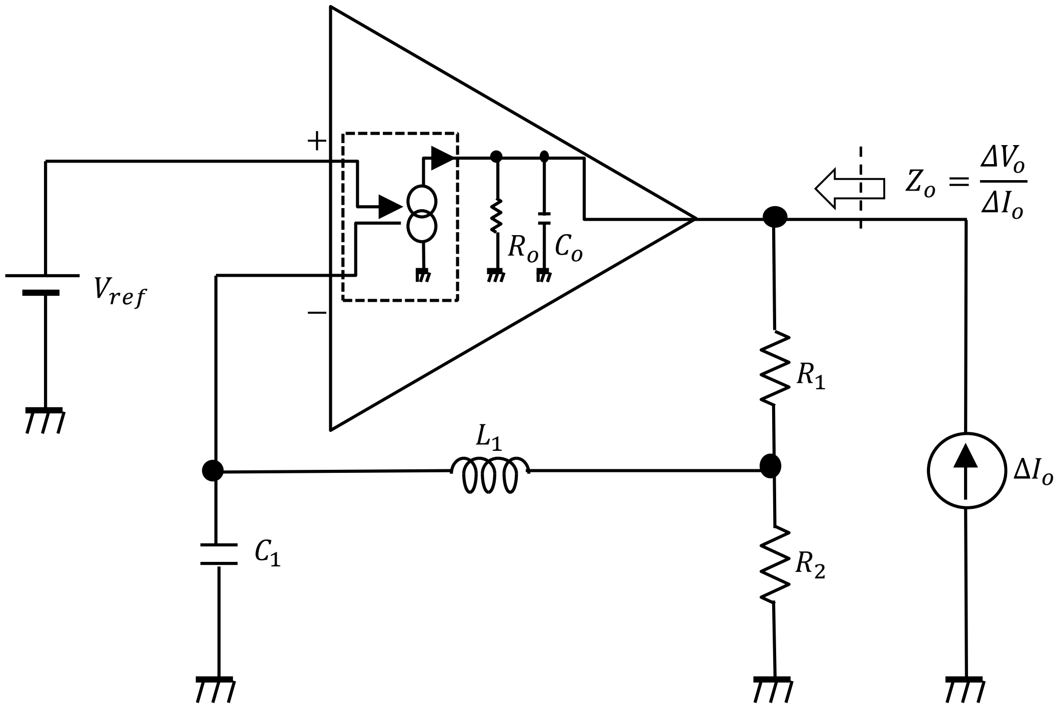
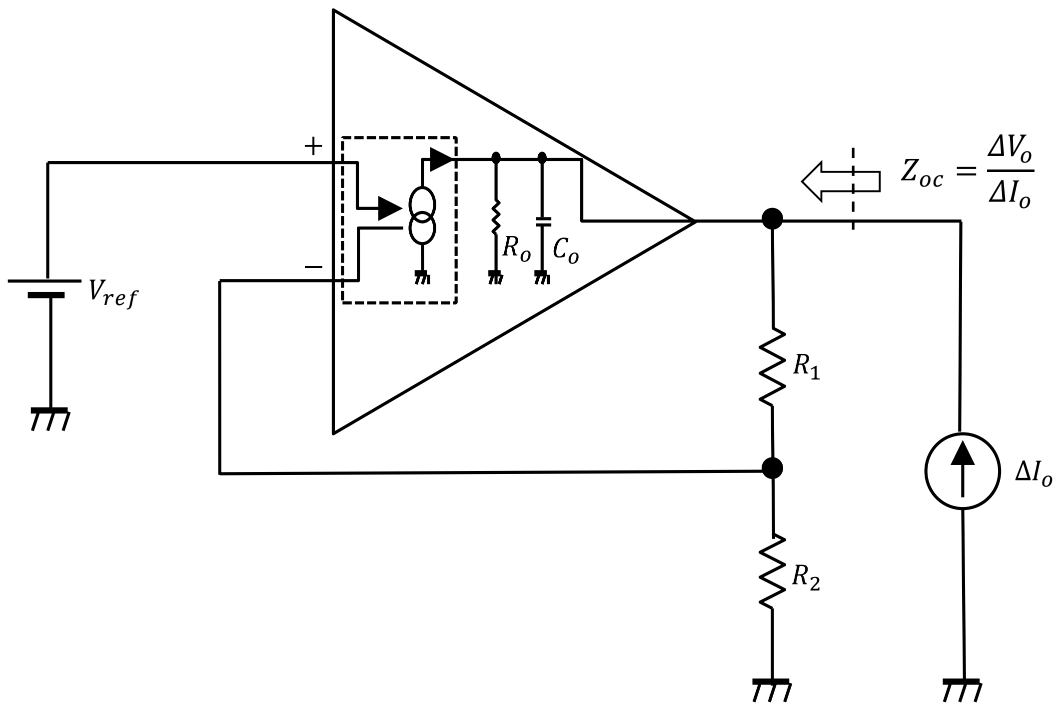


Fig. 7. Simulation circuit of the conventional loop gain measurement



(a) Measurement of Z_O

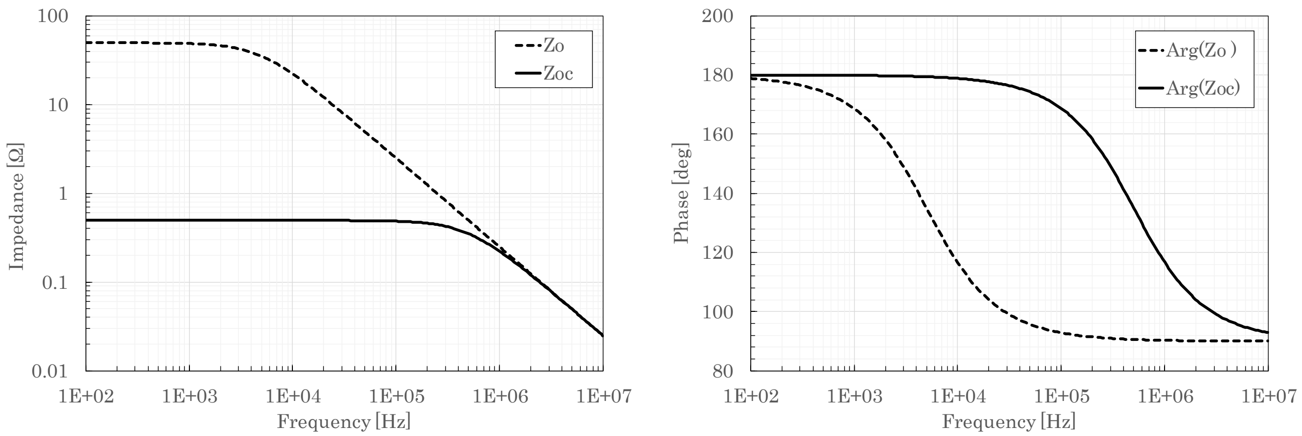


(b) Measurement of Z_{OC}

Fig. 8. Simulation of the proposed loop gain measurement.

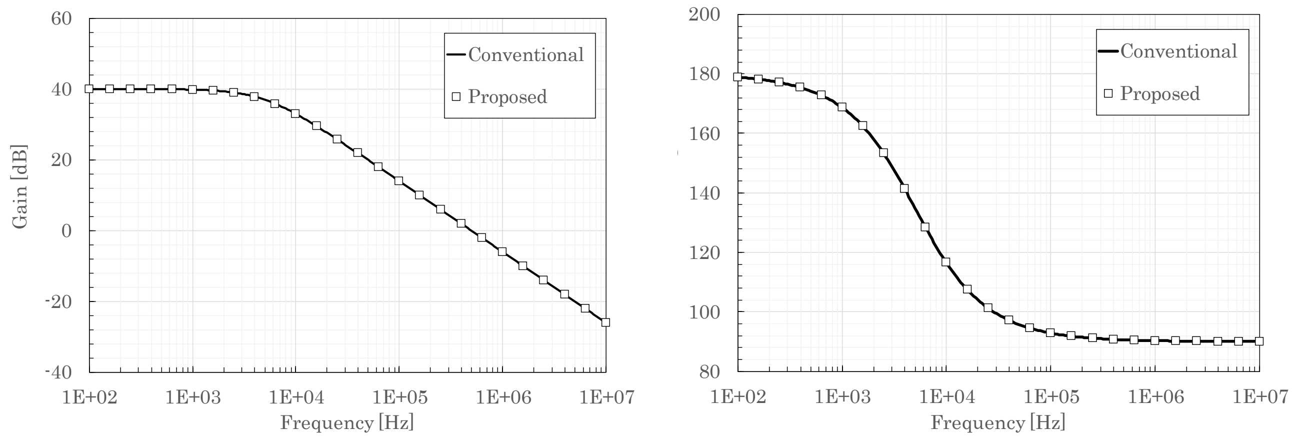
4.2 Simulation Results

Figure 9 shows the simulation result of the output impedance. The results with simulated loop gain measurements by the voltage injection method (conventional method) and the proposed method are compared in Fig. 10. The loop gain obtained from the proposed method is calculated by the output impedances using Eqs. (10) and (11), and we see that the loop gain obtained from the proposed method exactly matches with that from the conventional method.



(a) Impedance magnitude (b) Impedance phase

Fig. 9. Simulation results Z_o and Z_{oc} in Fig.8



(a) Gain. (b) Phase.

Fig. 10. Simulation result comparison of the loop gains obtained with the conventional and proposed methods.

5. Conclusion

We have proposed an improved measurement method of the loop gain of analog circuits with negative feedback configuration, and verified that this is applicable to operational amplifier circuits. By considering the derivation using the equivalent circuit model, it was confirmed that the proposed method was established at the circuit level. Our simulation results showed that the loop gain obtained from the proposed method exactly matches with the one from the conventional method. The advantages of the proposed method are that the loop gain measurement without breaking the feedback loop is possible, and also there are no circuit characteristics restrictions which the conventional method has at the signal injection point. The measurement of Z_o in Fig. 5 uses a large inductor as well as a large capacitor, and this configuration would need to be revised, which is under investigation.

Our method can also be applied to any circuit as long as it employs a negative feedback circuit. In future work, we hope to demonstrate the effectiveness of the proposed method for various circuits, such as LDOs, DC-DC boost converters and DC-DC buck-boost converters with simulations and experiments.

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