Journal of Technology and Social Science (JTSS)

Linear Vibration Analysis of a Thin Shell-panel Including Clamped Edges

Shinichi Maruyama [1,a], Daisuke Kumagai [1,b], Takao Yamaguchi [1,c] and Ken-ichi Nagai [1,d]

[1] Division of Mechanical Science and Technology, Faculty of Science and Technology, Gunma University, 1-5-1 Tenjin-cho, Kiryu, Gunma 376-8515, Japan

[a]<maruyama@gunma-u.ac.jp>, [b]<t161b021@gunma-u.ac.jp>, [c]<yamagme3@gunma-u.ac.jp>, [d]<kennagai@gunma-u.ac.jp>

Keywords: vibrations of continuous system, plate and shell structure

Abstract. In this paper, a new analytical method on natural frequencies and vibration modes of a thin shell-panel with clamped edge is presented. At the boundaries of two opposite edges, the shell panel is simply supported, and at the other two edges the panel is simply supported or clamped. Coordinate function of deflection is assumed with the product of power series and trigonometric function. Neglecting the effect of in-plane inertia force, the Donnell type equations are applied as the governing equation of the shell-panel. Stress function is obtained which satisfies exactly the compatibility equation with the inhomogeneous term due to the coupling with deflection and the in-plane boundary conditions. Applying Galerkin method to the equation of motion, natural frequencies and vibration modes are calculated and compared with the exact results.

1. Introduction

Nonlinear vibrations of a simply supported shell-panel have been analyzed by many researchers. The author [1] has analyzed assuming the deflection with product of sinusoidal functions in both in-plane directions and introducing in-plane stress function which satisfies compatibility equations. However, this procedure can not be applied to a shell-panel which included clamped edge. Nonlinear vibrations of a shell-panel with clamped edges have been analyzed introducing mode expansion of in-plane displacement [2]. Generally, higher number of terms is required in the expansion of in-plane displacement to express nonlinearity of the shell-panel accurately. Therefore, a new analytical method on natural frequencies and vibration modes of a thin shell-panel with clamped edge is presented in this paper. Neglecting the effect of in-plane inertia force, the Donnell type equations modified with lateral inertia are applied as the governing equations of the shell-panel. The response of lateral deflection is assumed with multiple modes of vibration. Stress function is introduced to satisfy compatibility equation and in-plane boundary condition. Applying Galerkin method to equation of motion, natural frequencies and vibration modes are calculated and compared with the exact result.

2. Procedure of Analysis

Fig. 1 shows the analytical model of the rectangular shell-panel in this research. All the symbols in the figure are shown with non-dimensional notations. We introduce the ξ, η axes along the in-plane directions of the shell-panel and the z axis in the lateral direction. The origin is taken at the center of the panel. The symbols α_x, α_y are non-dimensional curvatures in ξ and η directions, respectively. The symbol w denotes the deflection of the panel and u, v are in-plane displacements in the ξ and η directions, respectively. The aspect ratio of length of edges is denoted by $\beta = a/b$. As shown in Fig. 2, we assume two cases of boundary conditions of the panel (i) : SSSC and (ii) : SSCC, where the edges denoted by the symbols S and C are simply supported and clamped, respectively.

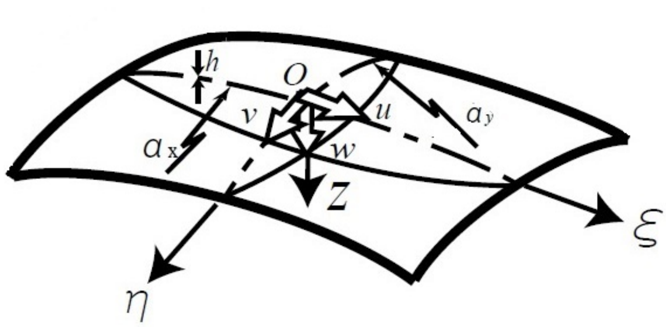


Fig. 1 Analytical model of a shell-panel

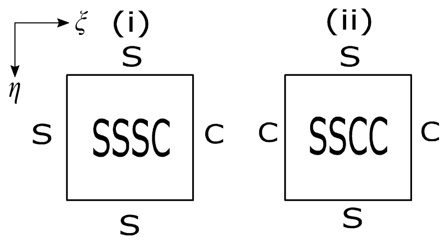


Fig. 2 Boundary condition

The Poison's ratio of the panel is denoted by ν. We assume the panel is sufficiently thin and that the in-plane inertia can be neglected, the equation of motion of the shell-panel is expressed as the following equations.

$$L(w, f) = w,_{\tau\tau} + \overline{\nabla}^4 w - \alpha_x \beta^2 f,_{\eta\eta} - \alpha_y f,_{\xi\xi} \tag{1a}$$

$$\overline{\nabla}^4 f = c\left(-\alpha_x \beta^2 w,_{\eta\eta} - \alpha_y w,_{\xi\xi}\right) \tag{1b}$$

Equation (1a) denotes the equation of motion of the panel in the lateral direction, and Eq. (1b) is the compatibility equation of the in-plane strain in terms of the stress function f, which is related to the in-plane resultant force n_x, n_y, n_{xy} as follows.

$$n_x = \beta^2 f,_{\eta\eta}\,,\ n_y = f,_{\xi\xi}\,,\ n_{xy} = -\beta f,_{\xi\eta} \tag{2}$$

The boundary condition (i) can be denoted by,

$$\xi = -1/2 : w = 0,\ w,_{\xi\xi} = 0,\ n_x = 0,\ v = 0$$
$$\xi = 1/2 : w = 0,\ w,_{\xi} = 0,\ u = 0,\ v = 0$$

$$\eta = -1/2, 1/2 : w = 0,\ w,_{\eta\eta},\ n_y = 0,\ u = 0, \tag{3}$$

while the boundary condition (ii) can be denoted by,

$$\begin{aligned} &\xi = -1/2, 1/2 : w = 0,\ w,_{\xi} = 0,\ u = 0,\ v = 0 \\ &\eta = -1/2, 1/2 : w = 0,\ w,_{\eta\eta},\ n_y = 0,\ u = 0. \end{aligned} \tag{4}$$

The deflection, satisfying the foregoing boundary condition for deflection, is assumed as follows.

$$\begin{aligned} &w\left(\xi,\eta,\tau\right) = \sum_m \sum_n \hat{b}_{mn}(\tau)\hat{\zeta}_{mn}(\xi,\eta) \\ &\hat{\zeta}_{mn}(\xi,\eta) = (d_4\xi^4 + d_3\xi^3 + d_2\xi^2 + d_1\xi + d_0)\cos[(m-1)\pi(\xi+1/2)]\sin[n\pi(\eta+1/2)] \end{aligned} \tag{5}$$

The notation $\hat{b}_{mn}(\tau)$ is unknown time function, and $\hat{\zeta}_{mn}(\xi,\eta)$ is the coordinate function for deflection which is the sinusoidal function in the η direction and is the product of power series and sinusoidal function in the ξ direction. The integers m, n are half-wave numbers in the ξ and η directions, respectively. The coefficients d_0, d_1, d_2, d_3 and d_4 are selected as follows to satisfy the boundary conditions.

$$\begin{aligned} &\text{(i)}\ \ SSSC : d_4 = 8, d_3 = 4, d_2 = -6, d_1 = -1, d_0 = 1 \\ &\text{(ii)}\ \ SSCC : d_4 = 16, d_3 = 0, d_2 = -8, d_1 = 0, d_0 = 1 \end{aligned} \tag{6}$$

The solution of the compatibility equation (1b) can be expressed as follows.

$$f = f_0 + f_1 \tag{7}$$

$$\begin{aligned} f_0 &= \frac{1}{2} p_y \xi^2 + \frac{1}{2} p_x \eta^2 + p_{xy}\xi\eta \\ &+ \sum_n \left\{ C_1 \cosh(\beta n'\xi) + C_2 \sinh(\beta n'\xi) + C_3 \xi \cosh(\beta n'\xi) + C_4 \xi \sinh(\beta n'\xi) \right\} \sin n'\overline{\eta} \end{aligned} \tag{8}$$

In the above expression, f_1 is the particular solution corresponding to the linear terms of deflection in the right-hand-side of Eq. (1b), which can be solved by equating coefficients. The notation f_0 is the homogeneous solution of Eq. (1b), where p_x, p_y correspond to uniform normal stresses and p_{xy} denotes uniform shear stress. The fourth term is introduced to have sinusoidal distribution of stress along η-direction which is needed to satisfy in-plane boundary conditions. These unknown coefficients can be exactly determined by the in-plane boundary conditions.

Then, substituting Eq. (5) and (7) to Eq. (1a) and applying the Galerkin procedure, the equation of motion is reduced to ordinary differential equations in terms of $\hat{b}$ in multiple-degree-of-freedom system.

$$\sum_m \sum_n \hat{B}_{rsmn} \hat{b}_{mn},_{\tau\tau} + \sum_m \sum_n \hat{C}_{rsmn} \hat{b}_{mn} = 0 \quad (r,s,m,n = 1,2,3,...) \tag{9}$$

Natural frequencies and eigenvectors are calculated with Eq. (9).

3. Results and Discussion

Analysis is conducted for cylindrical a shell-panel with the parameters $\beta = 1.5, \nu = 0.33, \alpha_x = 10$, $\alpha_y = 0$. In the analysis, the maximum number of the half-wave m, n is chosen to be 2.

Tables 1 and 2 show the results for the shell-panels with the boundary conditions (i) SSSC and (ii) SSCC, respectively. In the table, natural frequencies obtained by the present approximated analysis, exact results of frequencies, errors and vibration modes are listed. The present results of natural frequencies agree very well with the exact ones with only small difference.

Table 1 Comparison between approximated solutions and exact solutions of natural frequencies and vibration modes (SSSC) $\alpha_x = 10, \alpha_y = 0$

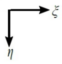
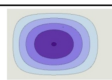
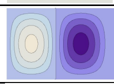
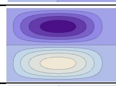
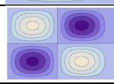
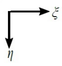
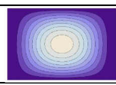
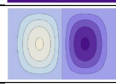
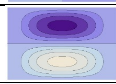
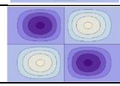
Mode (m,n)	Approximate solution	Exact Solution	Error [%]	Modal pattern 
(1,1)	43.051	43.0431	1.835×10^{-2}	
(2,1)	71.8615	71.6909	2.380×10^{-1}	
(1,2)	104.745	104.617	1.224×10^{-1}	
(2,2)	136.71	135.896	5.990×10^{-1}	

Table 2 Comparison between approximated solutions and exact solutions of natural frequencies and vibration modes (SSCC) $\alpha_x = 10, \alpha_y = 0$

Mode (m,n)	Approximate solution	Exact Solution	Error [%]	Modal pattern 
(1,1)	48.6599	48.6513	1.768×10^{-2}	
(2,1)	81.5647	81.4288	1.669×10^{-1}	
(1,2)	106.966	106.679	2.690×10^{-1}	
(2,2)	143.25	142.444	5.658×10^{-1}	

4. Conclusion

Analytical results are presented on linear vibration analysis of a thin shell-panel including clamped edge. In the analysis, coordinate function of deflection is assumed with the product of power series and trigonometric function and stress function is obtained which satisfies exactly the compatibility equation with the inhomogeneous term due to the coupling with deflection and the in-plane boundary conditions. Fairly good agreement is found in the comparison between the natural frequencies obtained by the present method and the exact ones, which verifies the present analysis.

References

[1] S. Maruyama, K. Nagai and Y. Tsuruta, "Modal interaction in chaotic vibrations of a shallow double-curved shell-panel", ***Journal of Sound and Vibrations***, Vol. 315, pp. 607-625, 2008.

[2] M. Amabili, "Effect of Boundary Conditions on Nonlinear Vibrations of Circular Cylindrical Panels", ***Journal of Applied Mechanics***, Vol.74, pp.645-657, 2007.

[3] S. Timoshenko and J. N. Goodier, ***Theory of Elasticity, 2nd edition***, McGRAW-Hill, 1951, pp. 46-47.